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THE UNIVERSITY OF ALBERTA

DEFLECTION STUDIES OF FLEXIBLE PAVEMENTS
IN ALBERTA

A DISSERTATION
SUBMITTED TO THE FACULTY OF GRADUATE STUDIES IN
PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE
DEGREE OF MASTER OF SCIENCE

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by

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ABSTRACT

Deflection characteristics of a pavement affect the permanence of a road structure. Deflection measurement under vehicle loadings can be obtained by various methods, but the advantages of the use of the Benkelman Beam have gained quick recognition because of its ease of operation, speed, and economy in obtaining deflection data. Because of the extensive use of the beam in pavement evaluation and the desire to correlate the beam data with other accepted test procedures for the purpose of the pavement design and construction control, analysis and correlation of the beam data with results from a theoretical approach are considered advisable. The investigations herein thus include interpretation and correlation of the beam data and theoretical analysis of the effects of the variables affecting deflection based on Burmister's theory. The Benkelman beam data were obtained from tests conducted by the Alberta Department of Highways.

Results from the studies show that the deflection of a pavement surface, measured in the fall, is closely related to its performance rating value, and that it constitutes an index of the ability of the pavement to carry loads without failure. In conjunction with Burmister's theoretical analysis, the beam data reflect the strength properties of a road structure and are valuable in pavement evaluation and construction control. Empirical equations expressing the relations between the variables affecting

The first part of the paper is devoted to a general discussion of the problem of the origin of life. It is shown that the problem is not only a scientific one, but also a philosophical one. The scientific aspect of the problem is concerned with the question of how life arose from non-life. The philosophical aspect is concerned with the question of whether life is a necessary part of the universe or whether it is a mere accident. The paper then proceeds to a discussion of the various theories of the origin of life. It is shown that the most plausible theory is that life arose from non-life through a series of chemical reactions. This theory is supported by the discovery of the RNA world and the discovery of the origin of the genetic code. The paper concludes by discussing the implications of the origin of life for the study of the universe and for the study of the human mind.

deflection have been established. Exponential relationships are found between the ratio of the moduli of the reinforcing layer and of the subgrade material, and pavement thickness, and between the deflection and the pavement thickness; deflection is found to vary proportionally with pavement temperature in the temperature range of 58 to 102°F. The applicability of the equations is subject to certain limitations, and further verification with more field data is therefore necessary.

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TABLE OF CONTENTS

	<u>Page</u>
ABSTRACT	iii
ACKNOWLEDGEMENTS	v
LIST OF FIGURES	viii
LIST OF PLATES	ix
LIST OF TABLES	xi
CHAPTER 1 - INTRODUCTION	1
CHAPTER 2 - LITERATURE REVIEW	4
CHAPTER 3 - THEORY	20
PAVEMENT DEFLECTION CONCEPTS.....	20
Variables Involved in the	
Determination of Deflection	20
Deflection Analysis --- Westergaard..	23
Deflection Analysis --- Burmister ...	23
PROPERTIES OF PAVEMENT COMPONENTS	
RELATED TO DEFLECTION	33
Subgrade	33
Base Course	36
Bituminous Surface	37
CHAPTER 4 - THEORETICAL ANALYSIS OF EFFECTS OF VARIABLES	
AFFECTING DEFLECTION BASED ON BURMISTER'S	
THEORY	45
THICKNESS AND STIFFNESS RATIO OF THE	
LAYERS	45
TEMPERATURE	54

TIRE PRESSURE AND CONTACT AREA	64
CONCLUSIONS	66
CHAPTER 5 - INTERPRETATION AND CORRELATION OF BENKELMAN	
BEAM DATA	68
DEFLECTIONS	71
Deflection Versus Thickness of Pavement	71
Deflection Versus Chronological Age of Pavement	83
Deflection Versus Subgrade Soil Type ...	89
Deflection Versus Temperature	93
PERFORMANCE RATINGS	95
Thickness of Total Pavement	97
Deflection	99
Coefficient of Variation of Ratings	99
CHAPTER 6 - CONCLUSIONS OF THE STUDIES AND SUGGESTIONS OF	
FURTHER INVESTIGATION OF THE TOPIC	108
BIBLIOGRAPHY	114
APPENDIX I - TABLES 1 to 8	121
TABLES 13 to 14	130
TABLES 16 to 24	133
TABLES 26 to 27	146
APPENDIX II - BENKELMAN BEAM DIAGRAM	154
ROAD MAP SHOWING THE HIGHWAYS INVESTIGATED	155
APPENDIX III - SAMPLE COMPUTATIONS	156
APPENDIX IV - APPLICATION OF STATISTICAL METHOD OF ANALYSIS	160

LIST OF FIGURES

<u>Figure</u>		<u>Page</u>
1	Boundary and Continuity Conditions of Stress and Displacement For a Two-Layer System	24
2	Influence Curves of the Settlement.. Coefficient, F_w , For A Two-Layer System	27
3a	Basic Pattern of Two-Layer Vertical Stress Influence Curve	29
3b	Basic Pattern of Two Layer Vertical Stress Curves	29
4	Evaluation of Vertical Stress and Shearing Stress Conditions at the Asphalt-Base Course Interface for A Three-Layer System	31

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LIST OF PLATES

<u>Plate</u>		<u>Page</u>
1	Effect of Modulus of Deformation of Pavement Material and Pavement Thickness on Deflection	48
2	Effect of Stiffness of Pavement on Deflection For a 15-inch Pavement Thickness	51
3.	Effect of $\frac{E_1}{E_2}$ and E_2 on Pavement Thickness Required For An Arbitrary Limiting Deflection of 0.05 inch	52
4	$\frac{E_1}{E_2}$ Versus Pavement Thickness Required for A Limiting Deflection of 0.05 Inch For Various Subgrade Modulii.	55
5	Effect of Change in Modulus of Deformation of Bituminous Material Due to Temperature Changes on Surface Deflection	57
6	Effect of Change in Temperature on Surface Deflection For Various Surface Thicknesses I	58
7	Effect of Change in Modulus of Deformation of Bituminous Material Due to Temperature Changes on Surface Deflection II	59
8	Effect of Change in Temperature on Surface Deflection For Various Surface Thicknesses II	60
9	Effect of Change in Temperature on Surface Deflection For Various Surface Thicknesses III	61
10	Effect of Tire Pressure on Deflection	65
11	Deflection Versus Pavement Thickness For The Four Highways	72
12	Evaluation of Approximate Modulii of Pavement Materials - I	76
13	Evaluation of Strength Properties of Subgrade and Reinforcing Layer	77
13a	Evaluation of Approximate Modulii of Pavement Materials - II	79
14	Deflection Versus Pavement Thickness	81

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15	Comparison of Deflections Measured In Accordance with CGRA and WASHO Benkelman Beam Procedures	84
16	Deflection Versus Chronological Age of Pavements - I	85
17	Deflection Versus Chronological Age of Pavements - II	87
18	Variability of Deflections For Old and Young Pavements	88
19	Deflection-Pavement Thickness-Pavement Chronological Age	90
20	Deflection and Subgrade Soil Type - I ..	91
21	Variability of Deflections For Pavements With Different Subgrade Soil Types	92
22	Deflection Versus Pavement Temperature	94
23	Pavement Thickness Versus Pavement Performance Rating	98
24	Deflection Versus Pavement Performance Rating - I	100
25	Deflection Versus Pavement Performance Rating - II	101
26	Deflection Versus Pavement Performance Rating - III	102
27	Deflection Versus Pavement Performance Rating - IV	103
28	Deflection Versus Pavement Performance Rating - V	104
29	Coefficient of Variation of Performance Ratings Versus Section Lengths of Pavements	106
30	Coefficient of Variation of Performance Ratings Versus Chronological Age of Pavements	107

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XXXXXXXXXXXXXXXXXXXXXXXXXXXX

LIST OF TABLES

<u>Table</u>		<u>Page</u>
1	Effect of Modulus of Deformation of Pavement Material and Pavement Thickness on Deflection	121
1a	Approximately Corresponding Values of Modulus of Deformation, Modulus of Subgrade Reaction, and CBR	122
2	Effect of Stiffness of Pavement on Deflection	123
3	Effect of $\frac{E_2}{E_1}$ and E_2 on Pavement Thickness Required For an Arbitrary Limiting Deflection of 0.05 Inch	124
4	$\frac{E_1}{E_2}$ and Pavement Thickness Required For A Limiting Deflection of 0.05 Inch and For Various Subgrade Modulii	125
5	Effect of Change in Modulus of Deformation of Bituminous Materials Due to Temperature Changes on Surface Deflection - I	126
6	Effect of Change in Modulus of Deformation of Bituminous Material Due to Temperature Changes on Surface Deflections - II	127
7	Effect of Change in Temperature on Surface deflection	128
8	Effect of Tire Pressure on Deflection	129
9	Summary of Information Regarding The Pavement Sections Studied - I	69
10	Summary of Information Regarding The Pavement Sections Studied - II	69
11	Summary of Information Regarding The Pavement Sections Studied - III	70
12	Summary of Information Regarding the Pavement Sections Studied - IV	70
13	Deflection-Pavement Thickness	130

1. The first part of the report is a general introduction to the subject of the study. It discusses the importance of the problem and the objectives of the research.

2. The second part of the report is a detailed description of the methods used in the study. It includes a discussion of the experimental design, the data collection procedures, and the statistical analysis.

3. The third part of the report is a presentation of the results of the study. It includes a discussion of the findings and their implications for the field of research.

4. The fourth part of the report is a conclusion and a discussion of the limitations of the study. It also includes a list of references and a list of figures and tables.

5. The fifth part of the report is a list of references. It includes a list of books, articles, and other sources used in the study.

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10. The tenth part of the report is a list of figures and tables. It includes a list of figures and tables used in the study.

11. The eleventh part of the report is a list of figures and tables. It includes a list of figures and tables used in the study.

12. The twelfth part of the report is a list of figures and tables. It includes a list of figures and tables used in the study.

14	Evaluate Approximate Modulii of Pavement Materials	131
15	Evaluation of Strength Properties of Subgrade and Reinforcing Layer	75
15a	Evaluation of Approximate Modulii of Pavement Materials	132
16	Deflection Measured in Accordance With CGRA Procedure	133
17	Comparison of Deflections Measured in Accordance With CGRA and WASHO Procedures	134
18	Deflection Versus Chronological Age of Pavement - I	135
19	Deflection Versus Chronological Age of Pavement - II	136
20	Variability of Deflections For Old and Young Pavements	137
21	Deflection-Pavement Thickness-Pavement Performance Ratings-Pavement Chronological Ages	139
22	Deflection and Subgrade Soil Type - I ..	143
23	Variability of deflections For Pavement With Different Subgrade Soil Types	144
24	Deflection and Pavement Temperature	145
25	Deflection Increase in Per Cent For A 4-Inch Asphaltic Material With Increasing Temperature	96
26	Deflection and Pavement Ratings	146
27	Coefficient of Variation of Rating-Section Length-Chronological Age of Pavement	148

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CHAPTER I

INTRODUCTION

Under a transient or a static load, a road structure deflects. The deflection mechanism in a flexible pavement is associated with elastic and plastic deformations which greatly affect the permanence, integrity, and stability of the pavement system.

The measurement of load-deflection characteristics has long been a controversial topic because of variations in methods, procedures, and the methods of interpretation of the test data. The data obtained from such tests are dependent on the structure as a whole, the portion tested and the nature of the applied load. This being so, the use of either the magnitude of the applied load or the allowable deflection value in taking account of dynamic, vibrational and repeated load effects and other controlling conditions, is mostly based on the correlations of the load test data with the performance of the pavement in service. In the widely used plate bearing test procedure, the applied loads are commonly either incremental, incremental-repetitional, accelerated, or repetitional (71)* with various unit pressures to produce deflections of 0.1 to 0.5 inch (27, 24) under the center of a rigid plate resting on a portion of a layered system; and the results may be either in terms of modulus

* Numerals in parenthesis refer to references contained in the bibliography.

of subgrade reaction or modulus of deformation of the material. While the inherent physical properties of the pavement material in the component parts have an important influence on deflection, the allowable deflection alone under a load system affects the strength criterion.

Deflection measurements under vehicle loadings can be obtained by various methods (18, 33, 39, 43, 54), and the Benkelman Beam, developed in 1953 by A.C. Benkelman of the Bureau of Public Roads, is a simple lever type deflectometer. ^{*} The beam measures the surface deflection under a loaded vehicle by observing the relative movements of the probe beam and the reference beam of the apparatus. The CGRA**Benkelman Beam Procedure measures mainly the elastic portion of the total deflection. Essentially elastic behavior exists in the subgrade and in the component parts of the layered structure of adequately designed pavements as reported by the WASHO Road Tests (46). The load required to produce a given amount of elastic deflection increases as the thickness of the layered system increases. Elastic behavior of the pavement is therefore maintained until the surface and the base course fail, if excessive consolidation and lateral displacement of the roadway material do not occur. Thus, the elastic deflection characteristic of the pavement is the controlling factor.

The dual wheel load of 9 kips with tire pressure of 80 psi in the CGRA Benkelman Beam Procedure (65, 69) corresponds to a

* See diagram in Appendix II.

** Canadian Good Roads Association.

circular plate of about 12 inches diameter based on prototype loading conditions. Over this circular area the tire pressure is assumed to be uniformly distributed.

The Benkelman beam data investigated in this dissertation were obtained from tests conducted by Alberta Department of Highways. The deflection measurements were mainly in accordance with the procedure specified by the Special Committee on Pavement Design and Evaluation of the CGRA. The purpose of the thesis is to review the significance of pavement deflection and the factors affecting the deflection in the light of Burmister's theoretical approach, which is based on the concept of a multi-layer elastic system. An attempt is made to correlate the available Benkelman beam data with the theoretical approach. Considerations are not given to stress distribution, durability, stability and other problems related to the design or evaluation of flexible pavement due to the limited scope of this thesis, rather than to an underestimate of their importance. Further research is needed in the development of the application of the beam data.

CHAPTER II

LITERATURE REVIEW

While the stress-deflection characteristics of rigid pavements have been under investigation at the major experimental concrete pavements since 1922 (in Bates, Pittsburgh, Arlington, and Maryland (61)), and while the theoretical analyses have been attempted since 1925 (by Westergaard (2, 31), Kelley, Pickett and others (35)), it was not until 1928 that intensive investigations as to the performance of flexible pavements in California were initiated by the California Division of Highways (33). The investigations indicated significantly that the elastic and plastic deformations and consolidation of the pavement materials contributed to the distress of flexible pavements (33).

On the test roads in California (33), deflection measurements under both static and moving wheel loads were carried out by the California Division of Highways and the Corps of Engineers in an effort at correlation with the performance of the pavements. Electric gauges for the measurement of deflection were employed. These were placed on the surfaces of the subgrade and of the pavement. Pavement deflections under moving wheel loads (speed about 10 mph) were measured; and in static tests, the deflections were observed when the movement of the surface under the load had apparently ceased. (The time was found to vary from 3 to 7 minutes

from the instant of application of the load.) Tests (18) showed that the effect of moving wheel loads on pavement structures was radically different from that of static loads. With sufficient load repetitions and with excessive deflections, serious failures in pavement developed rapidly, while several million load repetitions were required to cause failure when deflections were limited to 0.02 or 0.03 inch under a wheel load of 10 kips (18). For airfield pavements, because of the lesser number of repetitions, deflections of approximately 0.05 inch were permitted in pavements of good quality and high flexibility. The allowable deflection under heavier wheel loads could be slightly larger because the larger contact areas produced flatter deflection curves in the pavements (33). Under 10-kip wheel loads, some plastic flow of the adobe clay subgrade was observed for pavement thicknesses less than 25 inches, and it was believed that additional traffic would cause progressive plastic deformation resulting in grooving and ultimate failure after a sufficient number of load repetitions (33). Thus, Porter found the permissible deflection under a moving wheel load to depend on the flexibility of the pavement, the type of deformation, the radius of deflection bowl, and the number of load repetitions (18).

Static circular plate bearing tests were also conducted to provide a basis for correlation with measurements under pneumatic wheel loads in the deflection observations. Generally, the bearing tests with footprint areas and contact pressures corresponding to

that of the tires used resulted in lower deflections than those obtained from the static wheel loads. It was believed to be due to the difference in effect between plate and tire loading. Under tire loads, the pavement could bend throughout the loaded area, whereas with rigid plates essentially all the bending of pavement and the base occurred outside the loaded area (33).

The electric gauge equipment to determine the deflection of a pavement under service conditions (18) consisted principally of an electric gauge head and an oscillograph. The gauge head, mounted in a pipe coupling with a reference rod, was grouted into a pavement at a point where deflection measurement was desired. The relative movement of the solenoid coil in respect to the iron plunger in the gauge head was reflected on the oscillograph. The maximum deflection which could be recorded and the accuracy of the measurement depended on the accessories used. Elastic deflection up to 0.1 inch with an accuracy of 0.001 inch could be measured, or a maximum deflection of 0.01 inch with an accuracy of 0.0001 inch could be obtained when an electronic amplifier was used.

In 1940, Hubbard and Field (13) recognized that excessive deflection of a pavement surface constituted the major cause of failure of a road structure, and proposed an empirical method of determining pavement thickness required to carry any desired unit load. The method was developed from load-deflection tests at a given rate of loading up to 0.5 inch deflection under different sizes of bearing plates both on a given soil and on a combination of soil

7

and different thickness of pavements. The supporting value of the material was taken as the value corresponding to a loaded area of 130 square inches and to a deflection of 0.5 inch under the simple static type of concentrated loading. A plot of the bearing values at various pavement thicknesses could be prepared, from which the thickness of pavement required for any given unit load could be obtained.

During 1945 and 1946 an evaluation of airport runways in Canada was conducted by the Department of Transport and reported on by McLeod (26, 30, 50). Extensive bearing plate tests were run and the loads supported on 30- and 12-inch diameter bearing plates at 10 repetitions causing 0.5 and 0.2 inch total deflections were proposed to be adopted for the design of airfields and highway pavements respectively. The use of these limiting deflections was considered to be the most reasonable, because the critical deflection for the subgrade under flexible pavements depended on the thicknesses and the qualities of the overlying base and surface courses, and on the radius of curvature of the deflection curve in the pavement under wheel loads. These deflection values were also considered to correlate best with the actual wheel loads of unlimited traffic that could be supported by pavements without failure in each case.

Load-deflection tests using different sizes of bearing plate on representative cohesive subgrade soils and surface material enabled plots to be made of the ratio of the load supported at one deflection to that supported at another deflection for bearing plates

of 30 and 12 inches diameter against perimeter-area ratio at deflections ranging from 0.05 to 0.7 inch. These plots enabled extrapolation of load-deflection data from one bearing plate size to other sizes and to other deflection values within the plotted ranges.

As indicated by load-deflection curves, McLeod found that the load carrying capacity of a bituminous surface was greater than that of the various types of base course per inch of thickness and that of subgrade. The ratio of carrying capacity of the bituminous surface to that of the base course was found to vary from about 1.5 for those made with liquid asphalt and soft asphalt cement to about 2.5 for well designed and properly constructed asphalt concrete and sheet asphalt.

Disregarding the actual supporting value of material, a straight line relationship on a semi-log plot of number of repetitions of a given load against deflection indicated the effect of repetition: the larger the number of repetitions, the larger the total deflection.

The WASHO Road Test (40, 46) considered the measurement of elastic deflection of a pavement surface to constitute an acceptable index of its ability to carry loads without failure. Deflection was recognized to be influenced by many variables, such as vehicle speed, temperature of the surface, load, moisture content of the top layers of basement soil, and the thickness and quality of the pavement.

Deflection was found to decrease as speed increased up

to about 15 mph, and the effect of speed on deflection was found to be pronounced in relatively thin pavements. Vehicle loads were found to have a direct influence on deflection, which increased as loads increased. The effect of temperature on deflection was found to be pronounced up to approximately 70°F. Moisture content, which affected the bearing strength of a subgrade, was found to influence the deflection. Close correlation was found between moisture near the soil surface and deflection, especially in relatively thin pavements. As reflected in deflection values and structural behavior of the surface, the pavement with thicker surfacing was found to possess more flexural strength, and that with paved shoulders offered better lateral support.

The ability of a pavement to withstand repeated loads was found to depend not only on the magnitude of the deflection, but also on the radius of the deflection bowl; the stiffer the structure, the less the deflection and the larger the radius. With relatively small deflections, a pavement might probably withstand a large number of load repetitions, as deflection was found to be approximately proportional to the logarithm of the number of repetitions of load required to cause failure of a road surface. Deflections measured at creep speed up to about 0.045 inch in warm weather, and 0.030 inch in cold weather were reported as not associated with structural failure of a road surface of the test sections.

Deflection measurement devices employed by the WASHO Road Test included electronic gauges and the original and the

modified Benkelman Beams. The electronic gauge consisted of a linear variable differential transformer (LVDT), which converted mechanical movement into a form of electrical potential to record the deflection. Perforated plates were installed on the subgrade, and the surface discs consisted of the transformer holder into which the LVDT was placed prior to measurement of deflections. Oscillator voltage was applied to the LVDT primary, and the differential secondary output voltage was fed to the first stage of the amplifier through a simple, adjustable attenuator network. The elastic deflections both within the pavement structure and at the surface of the pavement could thus be measured in a recording oscillographic equipment. Because of the tedious process involved in installing each time when deflection readings were taken, and because of difficulties in the linear calibration, linear potentiometers were employed with minor modifications of the mechanical installation in place of the differential transformer.

The Benkelman beam is a simple lever type pavement-deflection indicator (39), which was first developed in 1953 by A.C. Benkelman of the Bureau of Public Roads. It consisted essentially of a narrow lever, a reference beam, and a dial micrometer. The narrow lever with a probe resting on the pavement surface was pivoted at the end of the reference beam. Movement of the probe beam with respect to the reference beam was measured by the dial micrometer.

Modifications of the original design (43) were made to improve its accuracy and to simplify its operation. The major

modifications included suspension of the probe arm below the datum beam and replacement of the supports with a shoe and an adjustable rear leg to improve the possibility of using the beam on rough or undulating pavement surfaces.

The deflection measurement by means of the beam (39, 43, 65) required the dial readings taken when the probe was inserted a distance of 4 feet 5 inches between dual tires of a rear wheel, when the rear axle was directly above the probe, and when the vehicle had driven away. The operation gave a true measurement of pavement deflection if the legs of the beam were outside the deflection bowl. Apparent deflections were measured when parts of the beam fell within the zone of significant influence, as reflected in the recording of positive or negative residual deflection depending on the degree of influence exerted on the legs and probe of the beam by the deflection pattern(65).

Developments and applications of the beam have been made (64, 65, 66) because of its advantages in operation and cost. The development of the beam was necessitated by the desire to correlate the data obtained therefrom with results obtained from other devices and to evaluate the load-carrying capacity of pavements for design and construction control purposes.

The use of the beam in conjunction with a Helmer Deflection Profile Recorder has been reported (65, 66). The recorder provided a virtual image of a longitudinal section through the pavement surface between the dual tires at the instant when they straddled and were

over the probe point. The arrangement of the arms of the device enabled obtaining a profile with a magnification of 10 times.

In Canada, extensive co-ordinated pavement evaluations on existing highways, constructed according to different field practices and subjected to various traffic and climatic conditions, have been carried out since 1958. Pavement deflections are measured by means of the Benkelman beam in accordance with the procedure specified by the Special Committee on Pavement Design and Evaluation of the Canadian Good Roads Association (65). The details of the beam remain the same as that used in the WASHO Road test, but the deflection measurements are taken under static wheel loads. In essence, the CGRA method of test requires the deflections to be recorded when the probe is between the dual tires and when the loaded vehicle is driven 8 feet 10 inches and 30 feet away from the probe; the readings are to be taken when the rate of dial movement is equal to or less than 0.001 inch per minute. The deflections thus measured correspond to the rebound portion of the deflection. The operation in most cases eliminates the possible interference of the probe and the tire walls as the vehicle is moved forward during the test, and minimizes the influence of the deflection pattern on the parts of the deflectometer.

Load-deflection studies at creep speed were conducted co-operatively by the Virginia Department of Highways and the Bureau of Public Roads (44) with two-axle truck of 9-kip wheel load equipped with 11 x 20 dual tires. The deflection of the entire pavement for

all test periods as measured by the Benkelman beam was 0.032 inch average. For any one test period the individual deflection value varied considerably between sections of the same overall thickness and even between points in the same section, the average range of variation in the former being about 0.025 inch, and, in the latter, about 0.015 inch.

Structural performance of flexible pavements in service was studied by means of load-deflection tests in Maryland (59). The deflection measurements as obtained by the Benkelman beam under a slowly moving wheel load of 11,200 lbs. ranged from 0.022 to 0.038 inch for pavement thicknesses ranging from 15 to 18 inches. The seasonal changes in deflection were found to be quite marked and consistent for older pavements but less so in the relatively recent sections. The residual value, which was the difference between the deflection and recovery measurements, generally ranged from 0 to 0.015 inch, approximately 15 to 25 per cent of the total deflection, and it tended to vary directly as the deflection. However, very few measurable amounts of permanent consolidation in the wheel paths were found, indicating that the deflection bowl pattern extended beyond the position of the front legs of the reference beam.

The magnitudes of deflections of a pavement of constant design were found to vary considerably even at the same site at different points. And in spite of the comparatively large deflections at some test sites for the spring test series, there was no evidence

of structural distress at these sites. No correlation between pavement deflection with the type and nature of the subgrade soil was found.

Hveem (48) in his analytical work showed that the primary factors affecting the elastic deflections of pavements were traffic load, resilience of underlying material, and the stiffness of the pavement structure. He found that for slow moving vehicles, the deflection exhibited a linear relationship to the load applied. While deflections under static loads were found to be always greater than the corresponding values under moving loads, the moving wheel load caused a sharp reversal of stress from compression to tension in every portion of the pavement in the wheel path. Pavement distresses are thus found to be due to the compression and rebound in the upper layers of the road structure as resulted from sharper bending and greater induced stress in the pavement slab. Consequently, the surface deflection characteristic is found to be significant and to be capable of ready correlation with the performance of the surface .

The deflections were found to be much smaller in pavements with thick gravel or compacted well-graded bases, just as in the case of a pavement with high slab strength. The large deflections of cracked pavements were ascribed to loss of continuity and of slab strength.

It was recognized that, as the failure of a pavement was a fatigue phenomenon and that cracking was the result of both the magnitude of bending and the number of repetitions, it was difficult

to specify a deflection value which an individual pavement could withstand under different traffic characteristics and climatic environments. However, in correlation with deflections measured at a wide variety of pavement types and conditions, allowable deflection of 0.05 inch for surface treatments and 0.012 inch for 8 inches plant-mix surface under 15-kip axle loads were suggested.

Work done at Purdue (53) included a series of strength tests on bituminous mixtures to obtain information on load-deformation characteristics of the material under various conditions of loading and temperature. The elastic deformations due to repeated loads were found to be independent of the number of repetitions, while the permanent deformation for each load cycle exhibited a decrease to a minimum point and then increased sharply leading to failure of the test specimen. The permanent deformation varied with the logarithm of the number of load repetitions, with straight line relationship up to a certain stage, beyond which the curve became abruptly steeper, indicating a rapid increase in the rate of deformation with increasing number of repetitions of load. The transition zone was found to be dependent on the magnitude of the applied stress. At a stress level approximately equal to 25 per cent of the maximum compressive stress under given conditions, termed as the endurance limit, there occurred very little increased permanent deformation upon load repetition. The endurance limit was affected by the rate of deformation and, especially, by temperature, and consequently the deflection characteristic of an asphalt mixture should also be affected by the

temperature, by the rate of applied load, and by the number of load repetitions.

To investigate the relationship between deflection, load repetition, and performance at various temperature, Monismith (60) used a spring base of specific stiffness to simulate the relative rigidity of the base-subgrade combination in an actual pavement structure. The deflected shape of an asphaltic beam on a specified spring base was found to vary with the magnitude of the applied load and with the number of load repetitions at a given temperature; the larger the load, the greater the deflection value; and, at a given load, the greater the number of repetitions, the larger the deflection measured. The effect of repeated flexing on the strength of the beam as measured by its modulus of rupture became pronounced when the number of repetitions was increased, as reflected in the decreased modulus of rupture and increased deflection of the beam. While there was little effect of asphalt content in the mixture on deflection, the beam of open graded mixture deflected more than one of dense graded mixture under the same load, and thus the load distribution characteristics and the magnitude of repeated deflection for a given base-subgrade combination and load were different in the two types of mixture. Increase in strength accompanied by an increased rigidity under dynamic loading with less deflection value was found for the specimen tested at 40°F. as compared to the specimen tested at 75°F. This phenomenon was believed to be partly due to change in viscosity of the asphalt as the temperature changed.

Like Westergaard (2, 3, 9, 31), Burmister (20, 62), and Palmer and Barber (12), Baker (67, 70) and Chandrangsu in Ohio State University utilized stiffness ratios of asphaltic beams and the supports as a fundamental factor in the investigation of the characteristics of asphaltic mixtures. The stiffness ratio was varied by employing asphaltic beams on leaves of different stiffnesses. Changing the span of the beam and of the steel leaf according to the desired deflection simulated the action of a subgrade behaving elastically beneath a flexible pavement. Static and dynamic load tests on cylindrical specimens as well as on beams proved that the modulus of elasticity in compression was a function of temperature and rate of deformation, and consequently the deflection was affected by these variables. At high rates of loading, the modulus tended to become a function of temperature only. Deflection was found to vary linearly with applied load, and, at a given load, the deflection decreased with increase in modulus of subgrade reaction. The linear function between logarithm of deflection and logarithm of load repetitions required to produce failure of a specimen indicated that the quality of pavement was one of the variables. This was in agreement with the findings of Hveem (48). The tests also showed that for any given deflection, a shallow beam withstood more load repetitions than a deeper one and that for both beams to withstand the same number of repetitions the deflection of the lighter beam was larger than that of the deeper one. At equal deflection, the deeper beam was actually subjected to a higher bending stress. Thus

it was pointed out that the stress and not the maximum deflection, controlled the performance of beams of different thicknesses.

From the foregoing review of the literature, our present knowledge of the deflection mechanism of pavements may be summarized as follows:

The deflection mechanism in a flexible pavement is associated with elastic and plastic deformations. With sufficient load repetitions and with excessive deflections, serious failure in pavement develops rapidly. The deflection of a pavement is influenced by the traffic load and the strength properties of the materials in the component parts of the pavement. The deflection characteristics of various types of pavement under different traffic and climatic conditions govern the allowable deflection that the pavement can withstand without failure. These include the flexibility of the pavement, the type of deformation, the radius of deflection bowl, and the number of load repetitions.

In the evaluation of the strength properties of the materials, expressed in terms of moduli of elasticity, it is found that the moduli values are dependent on temperature, rate of loadings, and the number of test load repetitions, and thus these variables, in turn, affect the deflection. The relative effects of the above variables on deflection are summarized as follows:

1. Deflection is found to vary linearly with applied load, and at a given load, the larger the number of load repetitions, the larger is the total deflection;

2. elastic deformation due to repeated loads is found to be independent of the number of repetitions but the permanent deformation is found to vary depending on the magnitude of the applied stress;
3. modulus value is found to increase with increasing rate of loading and with decreasing temperature, and at high rates of loading, the modulus tends to become a function of temperature only.

Electronic gauge and Benkelman Beam are the two devices mentioned in the measurement of pavement deflection. The basic function of the gauge is to convert the mechanical movement into a form of electrical potential so as to record the pavement deflection. The beam is a simple lever type deflectometer which measures the surface deflection under a loaded vehicle by observing the relative movements of the probe beam and the reference beam of the apparatus. The loaded vehicle may be in static or slowly moving condition depending on the deflection test procedure, as the CGRA or WASHO. The CGRA procedure normally measures the elastic portion of the total deflection of a pavement.

CHAPTER III

THEORY

PAVEMENT DEFLECTION CONCEPTS

The findings from the Stockton test road show that the deflection is 99.9 per cent elastic (32), and the results from the WASHO Road test (46) substantiate the elastic deflection behavior of the pavement for a transient and single application of wheel load on the mature pavements. To this extent, the use of the theory of elasticity is a rational approach in the solution of problems concerned with flexible pavement. With regard to time-dependent or repeated wheel load effects the quasi-elastic* character of the pavement material may still be analyzed by this theory if proper deformation constants are utilized (16). Burmister (20, 62) contributes to the development of such an approach through his layered system analysis.

Variables Involved in the Determination of Deflection

It has long been recognized that the strain in an elastic material is proportional to the stress. Boussinesq's solution for the elastic strain due to induced triaxial stresses under the center

* In a truly elastic material, there exists a constant ratio between the applied stress and the induced strain; if this ratio is not a constant but varies continuously over the entire range of stress, the material is classified as inelastic. However, if the ratio has an approximately constant value over a small but definite range of stress values, the material is said to be quasi-elastic and is considered as having essentially elastic behavior over this range of stress.

of a loaded plate indicates that the deformation is dependent on Poisson's ratio, the modulus of elasticity, the applied unit pressure on the plate, the radius of the plate, and the depth-radius ratio ^{1*}. Westergaard (2, 3, 9, 31) employs a modulus of subgrade reaction in the solution of the problems of stress and deflection at the surface of a structural slab. The deflection under the center of a load at the interior of a slab is affected by the modulus of subgrade reaction, radius of relative stiffness, and the ratio of the radius of the plate to the radius of relative stiffness ^{2*}. Burmister's layered system includes two parameters: the ratio of the radius of bearing area to the thickness of the upper layer, $\frac{r}{h}$, and the ratio of the modulus of the subgrade to that of the upper layer, $\frac{E_2}{E_1}$. These two ratios together with the rigidity of the supporting subgrade define the deflection at the surface of the system directly under the center of a load on a circular plate ^{3*}.

Deflection Analysis -- Westergaard

Following Winkler's theory of "beam on elastic foundation", Westergaard assumes that a slab is continuously supported by a subgrade of known strength. The subgrade strength is defined as the ratio of the unit reaction at any point in the tire contact area to the resultant deflection at that point. On the assumption that the slab is supported by a dense liquid, this value, known as the modulus

* See Plate 1a at the end of this chapter.

of subgrade reaction, is constant^{4*}. Consequently, there is no shearing stress in the subgrade. For actual subgrade material, the shearing resistance alters the theoretically assumed phenomenon of the uniform distribution noted for a dense liquid, and the maximum deflection occurs at the center of the loaded area with the magnitudes decreasing towards the edges of the area.

According to the assumed elastic medium on which the elastic beam is resting, the equation of the deflection curve of the beam due to a "uniform" unit pressure reaction is derived.^{5*} The term EI in the equation is analogous to the flexural rigidity of a plate (5), and this flexural rigidity is a function of the modulus of elasticity, Poisson's ratio, and the thickness of the plate. In the application to the analysis of stresses and deflection of a slab, recognizing that the resistance to deformation depends on the stiffness of the supporting medium and on the flexural stiffness of the slab, Westergaard adopts a "radius of relative stiffness"^{6*} to measure the stiffness of the slab relative to that of the subgrade. It is seen that the radius of relative stiffness has lineal dimension, that it depends on the properties of both the slab and subgrade: the stiffer the slab and the less stiff the subgrade, the greater is the radius of relative stiffness.

Since the subgrade does not behave as a dense liquid, the modulus of subgrade reaction is not a constant and, for the same unit pressure on a plate it varies with the size of the loaded

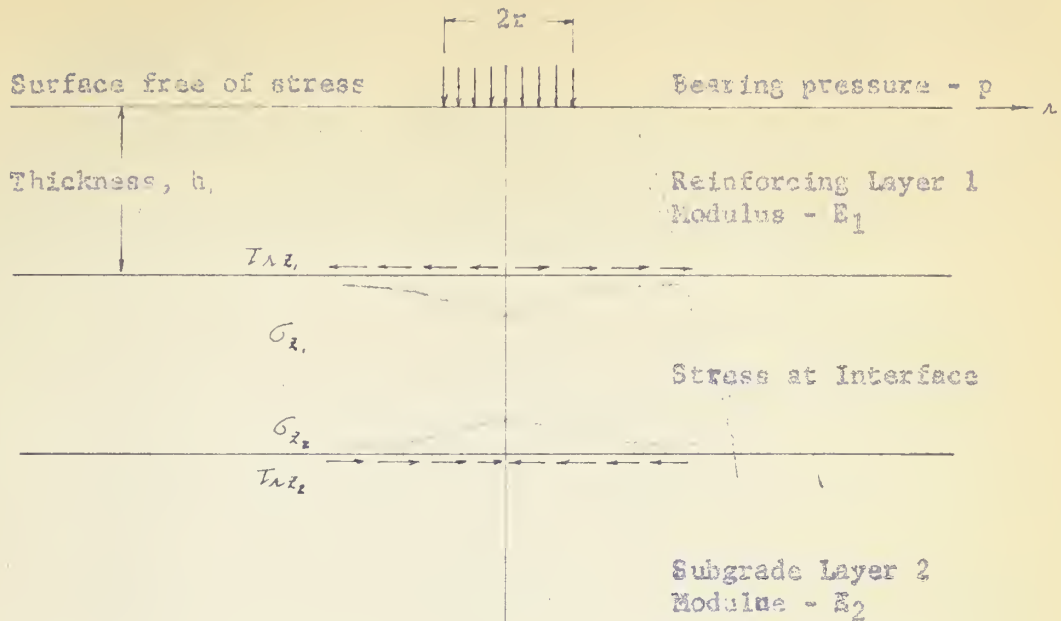
*See Plate 1b at the end of this chapter.

area, the thickness, and the properties of the subgrade soil, as well as with the amount of deflection. It is taken as the slope of the stress-deformation curve over the first 0.05 inch deflection (21, 28, 56). This curve is normally obtained by loading a steel plate of 30 inches diameter in the field. Investigators (22, 56, 63) show that the 'k' value does not vary appreciably with the diameter of the plate for plate diameters above 30 inches. The 0.05 inch deflection is assumed to be within the elastic range of the subgrade soil although this amount of settlement can be outside the elastic range of many soils. The 'k' value can also be estimated from laboratory tests with the modulus of elasticity of the material determined by triaxial compression tests and from Timoshenko's expression $(5, 7)^{8*}$ for the surface deflection of an elastic solid when loaded with a rigid circular disc.

Deflection Analysis -- Burmister

Based on the equations of elasticity for the three-dimensional problem of axial symmetry, Burmister (20) has developed a mathematical solution for the deflection of a pavement considering a total load uniformly distributed over a circular area on the surface of a layer of elastic material, which in turn rests on a semi-infinite elastic body. To satisfy the equilibrium and compatibility conditions, Burmister's stresses and deflection equations are based on certain essential assumptions regarding boundary and continuity conditions as shown in Figure 1.

*See Plate 1b at the end of this chapter.



Boundary Stress and Displacement Conditions:

Surface - Free of stress outside of loaded area $\sigma_z = \tau_z = 0$

At $z = \infty$ $\sigma_{z_2} = \sigma_{x_2} = \tau_{xz_2} = 0$ $w_2 = u_2 = 0$

Continuity Stress and Displacement Conditions at the Interface:

Normal Stresses	$\sigma_{z_1} = \sigma_{z_2}$	
Shearing Stresses	$\tau_{xz_1} = \tau_{xz_2}$	
Vertical Displacements	$w_1 = w_2$	z - direction
Horizontal Displacements	$u_1 = u_2$	x - direction

Note : There is a discontinuity in σ_x across the interface because with displacements $u_1 = u_2$, the stresses σ_{x_1} and σ_{x_2} will be determined by the moduli E_1 and E_2 , respectively, of the two layers.

Figure 1 Boundary & Continuity Conditions of Stress and Displacement For A Two-Layer System
(From Ref. 20)

1. The material in each layer is homogeneous, isotropic, and elastic;
2. The surface layer is weightless and infinite in extent in the horizontal direction only but the underlying layer is infinite in extent horizontally and vertically downward;
3. The surface layer is free of normal and shearing stresses outside the limits of the loaded area. The stresses and displacements in the subgrade are each equal to zero at infinite depth; and
4. The materials are fully continuous with no relative movement at the interface, and there is no shear stress acting across the interface in the case that it is frictionless.

According to Burmister, the load-settlement characteristics of a layered system are influenced by the ratio of the radius of bearing area to the thickness of the surface layer, the ratio of the modulus of deformation of the subgrade to that of the surface layer, and the rigidity of the supporting subgrade itself. Based on the above-mentioned assumptions and variables involved, the deflection at the surface of a two-layer system directly under the center of a load applied uniformly through a circular bearing area was given by Burmister as:

$$\Delta = \frac{1.5 \text{ pr}}{E_2} F_w$$

The above formula is simply the Boussinesq's settlement equation with

a multiplying coefficient F_w , a function of $\frac{r}{h}$ and $\frac{E_2}{E_1}$. This coefficient has two outer limits. With an extremely thick surface layer, the coefficient numerically equals $\frac{E_2}{E_1}$, as $\frac{r}{h}$ approaches zero. In this case the deflection is only governed by the rigidity of the surface layer, as

$$\Delta = \frac{1.5 \text{ pr}}{E_1}$$

At the other limit, the coefficient becomes equal to unity, as $\frac{r}{h}$ approaches infinity resulting from either a large value of 'r' or a very small 'h'. In this case, the settlement equation is practically the same as Boussinesq's. Thus, F_w describes the load-deflection responses of a two-layer system. While the 'r' in the $\frac{r}{h}$ ratio is a measure of the concentration of the load which affects the stress and deflection, the 'h' influences the bending moment and stress intensity and thus affects the magnitude of deflection and the shape of a deflection curve.

Figure 2 discloses the implicit dependence of the settlement coefficient for surface deflections on the basic layered system parameters. For a constant $\frac{h}{r}$, deflection decreases as stiffness ratio of the layered system increases, as indicated by a decreasing value of the coefficient. For a constant 'r', increase of the thickness of the system from some minimum value toward a value corresponding to $\frac{h}{r} = 1.0$ or greater results in decrease of deflection, but the effect of increasing in thickness on deflection becomes less pronounced.

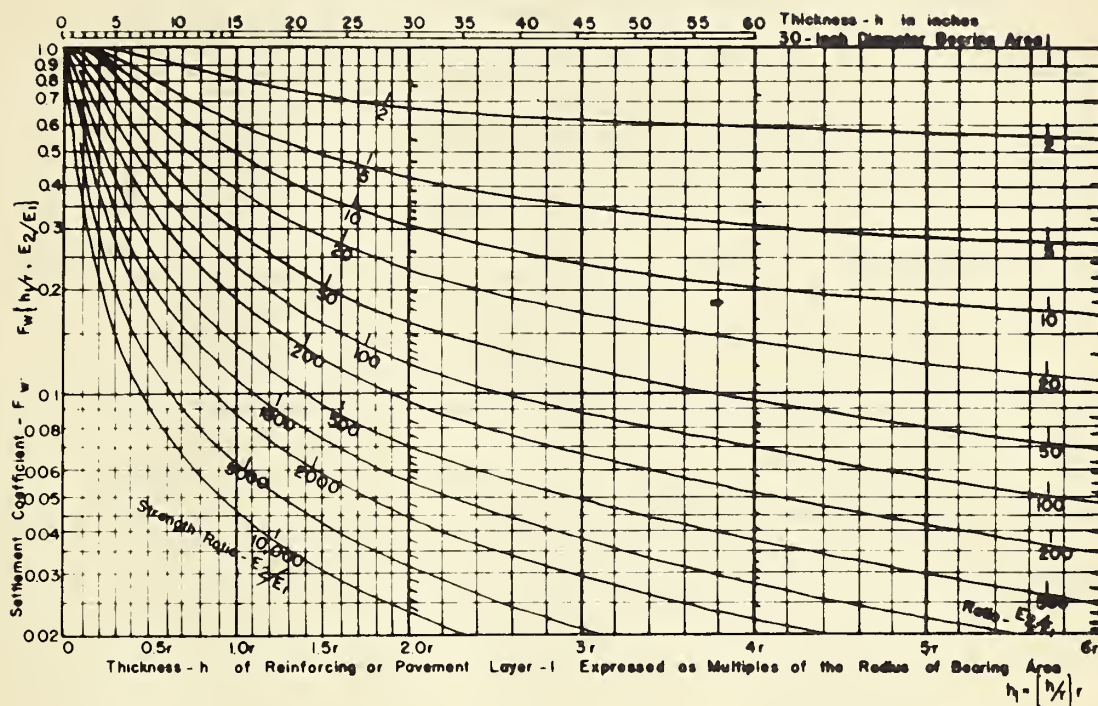


Figure 4. Influence curves of the settlement coefficient, F_w , for the two-layer system; basic load-settlement relation: $w_c = 1.5 \frac{p r}{E_2} \cdot F_w$

$$F_w = \frac{w E_2}{1.5 p r}$$

$$\mu_1 = \mu_2 = 0.5$$

Figure 2
 (From Ref. 62)

To understand the essential nature of the mechanics and effectiveness of the layered system, an analysis of the stress distribution and the deflection at the surface and layer interfaces are necessary.

Vertical stresses --- The distribution of vertical stresses is greatly influenced by the surface layer which ordinarily has better strength properties than the subgrade. With a surface material having a modulus 100 times that of the subgrade, the vertical stress under the center of a circular plate for the two-layer system at the base-subgrade interface is about 10 per cent of the applied pressure, while that based on Boussinesq, with the same modulus of elasticity in both base and subgrade materials, is about 70 per cent (62). The stress gradients at or near the interface are vastly different in both analysis, although these stress gradients approach a common level at great depth (62). The above analysis is based on the ratio of radius of the loaded area to the thickness of the reinforcing layer equal to unity. As the ratio increases, the effectiveness of the layered system in reducing stresses imposed on the subgrade layer is decreased, but at a less rapid rate for the larger strength ratio, $\frac{E_1}{E_2}$. The above discussions are illustrated in Figures 3a and 3b.

Shearing stresses --- Due to the vertical stress gradient and to the continuity at the interfaces, the shearing stresses induced in the interface region of a layered system are important and critical, compared to those evaluated by Boussinesq's equation



THE HISTORY OF THE

REIGN OF KING CHARLES THE FIRST

BY SAMUEL JOHNSON

LONDON

Printed by J. B. G. & Co. 1795

IN TWO VOLUMES

VOLUME THE FIRST

CHAP. I. OF THE REIGN OF KING CHARLES THE FIRST

IN THE YEAR 1625

THE KING WAS CROWNED AT WESTMINSTER

ON THE TWENTY-NINTH OF FEBRUARY

IN THE PRESENCE OF THE LORDS OF THE PARLIAMENT

AND OF THE COMMONS

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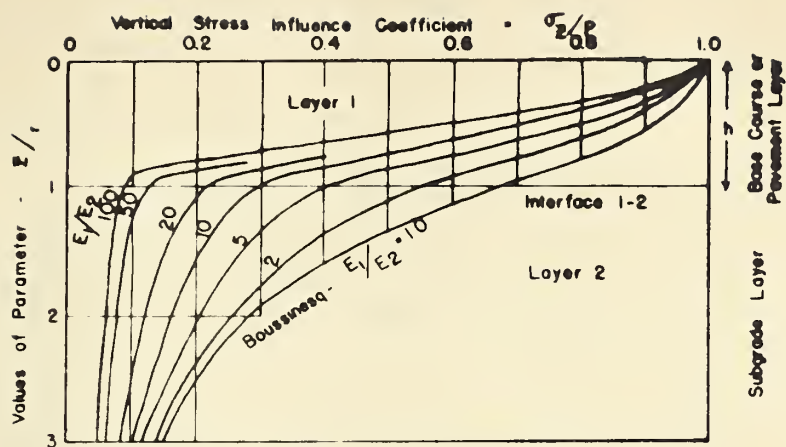
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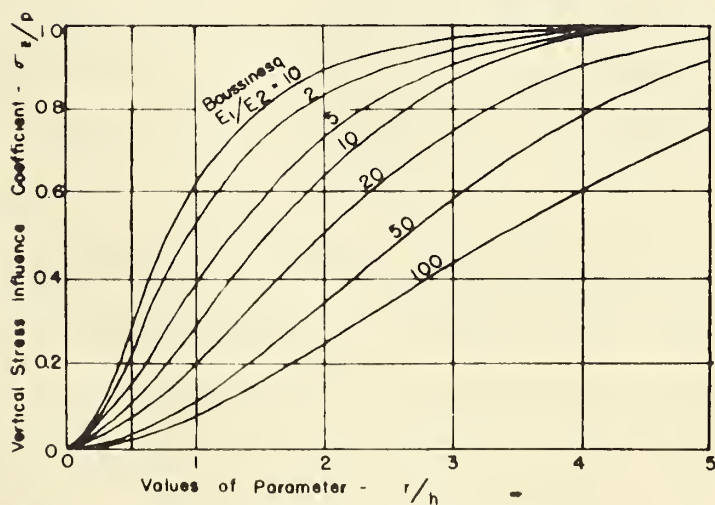
ON THE TWENTY-NINTH OF FEBRUARY

IN THE PRESENCE OF THE LORDS OF THE PARLIAMENT

AND OF THE COMMONS



a). Basic Pattern of Two-Layer Vertical Stress Influence Curves (σ_z/p vs z/r) for $r/h = 1.0$ and $\mu_1 = \mu_2 = 0.5$



b). Basic pattern of two-layer vertical stress curves (σ_z/p vs r/h) at the interface - $z = h$. $\mu_1 = \mu_2 = 0.5$

Figure 2. Effectiveness of two-layer systems in reducing vertical stresses imposed on the subgrade layer; Burmister problem; Fox stress influence coefficients (Ref. 3).

Figure 3a & 3b
(From Ref. 62)

at the same depth in a homogeneous deposit, as disclosed in Figure 4. At the interface of a layered system, the induced shearing stresses are more than four times that in a homogeneous deposit at the same depth, and they are equal in magnitude to the vertical stresses in the first case but are only 17.5 per cent of the vertical stresses in the second case.

Deflection --- The stiffness of the surface layer and the continuity characteristic, which enables the system to sustain relatively high shearing stresses at the interface, results in a large decrease in deflection in the system. The stiffness can be achieved by employing good quality materials having better strength properties or by thickening the layers. Since the effect of thickening becomes less pronounced as the thickness of the layer is increased, more marked and effective improvement can be achieved by using a material having better strength properties and by attaining a full continuity in the layer interfaces through construction procedures (20). The desired effect is one of reducing the magnitude of deflection to such a value under design wheel loadings that the shearing stresses induced in the interfaces are well below critical values and that the accumulated inelastic shearing strains do not affect the performance of the pavement during the expected life of the pavement.

Extension of Burmister's analysis has been carried out by Southwell, Fox, Wilson and Williams (34). Burmister evaluated the WASHO Road test results by the layered system and concluded that

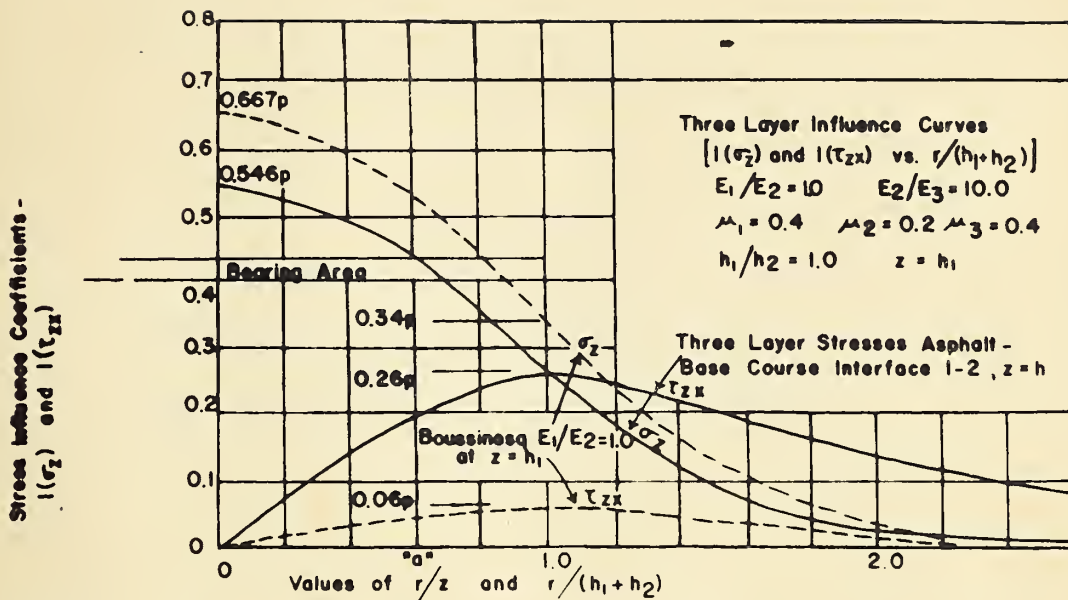


Figure 3. Evaluation of vertical stress and shearing stress conditions at the asphalt-base course interface for a three-layer system with regard to relative magnitudes and distribution on the interface, and to the potential critical conditions in comparison with the Boussinesq stresses for a homogeneous system; Burmister problem (Ref. 4).

Figure 4
 (From Ref. 62)

the correlation between the field measurements and the theoretical analysis was good.

In the study of the materials presented above, the assumption made by Westergaard that the reactions of the subgrade are vertical only and are proportional to the deflection of the slab is found to be open to criticism. This assumption implies that soil has no shearing resistance, and the slab, considered to rest on a series of independent closely-spaced springs, receives full support from the subgrade. These apparent weaknesses of the assumption lead to the belief that Burmister's approach may have more realistic value for the analysis of the deflection characteristics of flexible pavements. The following theoretical analysis in this thesis is thus based on Burmister's theory.

PROPERTIES OF PAVEMENT COMPONENTS RELATED TO DEFLECTION

The load-carrying capacity of a flexible pavement is brought about by the load-distributing characteristics of the layered system consisting of a subgrade, a base course, and a wearing surface. The component parts are inter-dependent and the characteristics of each affect the deflection characteristics of the structure.

Subgrade

The subgrade, normally constructed from the natural soil, is the foundation upon which the pavement rests. Because the subgrade supports transmitted wheel load (6, 50, 62) and has more influence than any other variable on flexible pavement thickness requirements (68), the different behaviour in deflection under different types of vehicle loads is important. A pavement may be considered to have failed when the permanent deflection of the underlying soil is of such a magnitude as to cause serious ruttings or cracks in the surface. Elastic deflection, however, may cause fatigue failure in the surface due to compression and rebound by the passing wheel loads, even though there occurs little plastic deflection in the subgrade (45). Repeated loading due to moving vehicles causes a subgrade soil to deflect more than a sustained loading from a static wheel load of equal magnitude (47). However, different soils under repeated load have different deflection characteristics, as reflected in the magnitudes of elastic and plastic deflections. Some soils having low resistance to plastic deformation may also exhibit high elastic deflections and some soils may exhibit extremely small plastic deflections and yet have high elastic deflections (49). For clays of

low to medium plasticity, density changes would have a great effect on total deflection, while having only a small effect on the elastic deflection.

The resilience of a subgrade soil depends on the basic physical properties such as cohesion, internal friction, and volume change of the soil due to changes in environments (63). The following is a general discussion of each of these factors as related to the deflection characteristics of the subgrade.

1. Cohesion and Internal Friction

The strength of a soil to resist deformation is derived from the combined effect of cohesion and internal friction, which are functions of the properties of the individual soil grains, the density, the imposed restraint, and of the moisture content. Soil particles vary in hardness, shape, angularity, and surface behaviour; and the density depends on the shape and gradation of the particles as well as the moisture content at a given compaction. The moisture content influences to a great extent the supporting value of a cohesive material but has only a negligible effect on a cohesionless soil; the supporting value of a cohesive material is almost entirely due to cohesion which is attributed to electrostatic forces of attraction between particles at their boundaries of contact with each other. Moisture plays interrelated roles: it supplies apparent cohesion through surface tension phenomenon and, at the same time, lubrication, depending on the thickness of the absorbed water films surrounding the individual grains. Therefore the strength of a subgrade not only depends on the soil type but also on

the field conditons under which it exists. A high subgrade support is attained from either high cohesion or high internal friction, or both. Clays with stiff consistency give high cohesion; well graded aggregates produce high internal friction; and well graded aggregates with a proper binder result in a desirable subgrade.

2. Volume Change

Swelling and shrinkage often occur in a compacted subgrade before or after the pavement is laid. The amount of volume change depends on the degree of consolidation, the moisture content employed in compaction, and especially on the structure of the subgrade soil. Most clays have a complex network of scale-like particles, and consist of chemically hydrated alumino-silicates, which are formed during the leaching processes. The surfaces of these scale-like particles carry negative electric charges which affect the arrangement of the bi-polar molecules in a liquid like water. The intensity of the charge depends to a large extent on the nature of the adsorption complex and the mineral character of the soil particles, which are responsible for the properties such as plastic yield, compressibility, elasticity, swelling and shrinkage.

Water molecules can be associated with the surfaces of soil particles and with the interstices or capillaries between particles. Variation in the former is a physico-chemical change, and the action of the latter is a physical one caused by surface tension forces having the character of an externally applied force. Swelling is associated with hydration of particles. The adsorbed water-films grow during the

wetting of a clay. This growth of water-films results in an increase in the total volume of the soil structure and therefore in swelling of the subgrade, causing a reduction in bearing capacity of the road structure, as reflected in large magnitude of deflection.

Shrinkage is caused by loss of moisture in the soil. This depends on the action of capillary forces as a result of the reduction of moisture content, and on the resistance furnished by the soil particles being consolidated. The theory of shrinkage explains the phenomenon: when a soil is completely saturated, the contractive force exerted by the surface tension of water is practically zero. As the soil moisture is decreased, the capillary tension on the outer surface of the sample exerts an allround compressive force on the soil particles, causing a contraction of the soil until the soil attains such a volume that the resistance of the soil to further reduction in volume just equals the capillary pressure exerted by the evaporating moisture (4). Shrinkage in the subgrade will result in differential vertical movement and cause permanent deformation in the roadway.

Base Course

The basic functions of a base course are to provide a stress-distributing medium to transmit the wheel load stresses to the subgrade without being subjected to shearing stresses greater than it can withstand itself, and to control the magnitude of distortion due to volume change in the subgrade. These basic properties are influenced by the gradation, particle shape, and relative density of the material. In an aggregate which contains little or relatively low percentage of

finer, the mechanical interlocking and the relatively rigid structural framework contribute to the stability of the mixture. The density and the frost susceptibility of the soil depend on the presence and quality of the fines. In a material which contains a great amount of fines and has no grain-to-grain contact of the coarse aggregates, the gross mixture possesses no structural framework; hence, the stability depends on the grading as well as plasticity which is a function of moisture content and the surface-chemical properties of the fines. Depending on the magnitude of imposed strain by vehicle load, this component part of a road structure possesses elastic properties to resist deformation. If the imposed strain is excessive due to single or repeated application of vehicle load, plastic deformation may result.

Bituminous Surface

An asphalt mix is composed of two essential parts, asphalt and aggregate. The load-carrying capacity is dependent mainly on the mineral aggregate which constitutes the entire framework and the major portion of the absolute volume of the mix. The function of the bituminous binder is to coat the aggregate with an adhesive ductile film of bitumen to attain sufficient mechanical stability, resiliency and flexibility so as to resist disruptive forces. Thus, structurally the deflection of a pavement depends on both constituents and on the characteristics of the mixture. Because of the rheological* and thermoplastic** properties of a bituminous binder, the properties related to the deflection characteristics of a road mix vary with time of loading

* Refers to the flow and deformation properties of matter.

** A material is said to be thermoplastic if it becomes more plastic with increasing temperature.

and temperature. These physical properties are discussed below:

1. Internal Friction --- Particle shape, surface texture, and grading.

These physical properties of aggregates govern the internal friction, the mechanical arrangement, and interlocking of the individual particles of the mass. The surface texture influences the aggregate strength in an asphaltic paving mixture (68). For two mixtures with the same film thickness and a given contact pressure, the one with irregular surfaced aggregate will develop greater frictional resistance and probably deflect less under loads. The particle shape and gradation influence the behaviour of a paving mixture during placing, compaction, and under service. In a dense-graded mixture, the coarse aggregate contributes to stability but the use of a relatively large proportion of it generally produces very high voids in the total system. The quantity and characteristics of fine aggregate and mineral filler control the percentage of voids in the total aggregate and affect the stability and the amount of bitumen which can be incorporated in the mixture. An open-graded mixture exhibits a better flexibility characteristic, because asphalt in the mixture would exist in thicker films even though smaller quantities of asphalt are used. However, it creates problem in selecting aggregates to insure adequate resistance to deformation under load. Durability is also a problem, because the open-graded mixture is more susceptible to weathering (60).

2. Cohesion

Cohesion in a bituminous mixture contributes to paving

stability, and the cohesive force is attributed to high viscosity of the asphalt present in the mixture. As associated with the effective thickness of asphalt film, the viscous resistance increases with decrease in film thickness to some optimum value which varies depending on types of asphalt (52). While thick films result in increased durability, they cause instability of a roadway in the form of waves and ruts by shoving due to the action of traffic in warm seasons. On the other hand, thin films may produce brittle mixtures which tend to crack and ravel excessively, and thereby shorten the life of a pavement. It is thought (53) that the elastic rebound takes place principally in the asphalt films while the permanent shear deformation is attributed to the reorientation of aggregate particles when the asphalt film has been reduced to critical thickness. Thus the asphalt film thickness controls the deflection characteristic of a paving mixture.

Viscous resistance of a given asphalt film varies directly with the rate of deformation and inversely with the temperature. It has been shown (60, 68) that the asphalt develops a correspondingly larger resistance to shearing deformation as the rate of load application is increased. Under static or low rates of loading, asphalt develops little shearing resistance and thus contributes little to the stability, resulting in plastic flow of the mixture. Under rapidly applied loads, considerable shear strength is developed. Similarly the tensile and compressive strength of the asphalt material increase with increasing rate of deformation, and increase with decreasing temperature (41), as reflected in a smaller magnitude of deflection under a given wheel load.

In the temperature range of $40 - 275^{\circ}\text{F}$, the semi-logarithmic plot of viscosity of bitumen against temperature is found to be linear (60). Due to the effect of temperature on viscosity, a paving mixture of high temperature susceptibility* gives smaller deflections at relative low temperatures, but it gives larger deflections at high temperature as compared with less susceptible material of the same hardness at the same temperature. The degree to which an asphalt softens excessively or becomes brittle over a range of temperature encountered is often responsible for pavement distress. The combination of excessive hardening at low temperature and lower subgrade support during spring thawing may result in pavement failure, because flexural stresses of high magnitude are induced in the asphalt surface as a result of a sharp deflection curve in the pavement under vehicle loads.

The variation of consistency of asphalt with time is of importance and is related to the durability of the asphalt. In general, the degrees to which an asphalt hardens in service is a function of the quality of the asphalt, of the mix design, and of the construction procedures. Age-hardening (55, 57) is the result of oxidation, volatilization, action of light, action of water, and of chemical changes occurring internally with time. These effects cause a reduction of resins and a relative increase in asphaltenes, with the non-polar oils in the colloidal system remaining substantially constant. This physical change in molecular sizes causes a loss of hydrogen due to the formation

* Refers to the property by virtue of which the consistency of an asphalt is affected by temperature.

of water, which affects the adhesion of asphalt. It also causes an increase in the consistency of the asphalt which results in a reduction of the ability to absorb the energy of imposed stresses by the process of deformation.

Summarising, we find that Westergaard's and Burmister's solutions for stresses and deflection of a pavement are based on the assumption that the materials are elastic to a certain extent. The basic difference between the two solutions lies in the assumed character of the supporting medium. Westergaard assumes that the subgrade behaves like a dense liquid and so the shearing strength of the subgrade material is neglected; consequently, there can be no shearing stresses at the pavement-subgrade interface. On the other hand, Burmister assumes that the component parts of a pavement structure are fully continuous at the interfaces; thus, the deflection at any point is not necessarily proportional to the stress at that point, but is affected by the pattern of stress distribution as a whole.

The stiffness ratios employed by both solutions have different physical forms and different magnitudes because the difference in vertical stress distribution under a rigid and a flexible slab affects the bending moments, stresses, and the deflections of the surface.

According to Burmister's analysis, the deflection of a pavement under wheel loads is influenced by the following variables :

- (i) pressure intensity,
- (ii) radius of contact area,
- (iii) subgrade modulus,

(iv) stiffness ratio, and

(v) radius of loaded area - pavement thickness ratio.

Deflection is also influenced by the properties of the component materials and the controlling conditions of service. The strength of a soil to resist deformation is derived from the combined effect of cohesion and internal friction which are functions of the properties of the individual soil grains, the density and the imposed restraint. These, in turn, are dependent on gradation and the moisture content of the soil.

The bituminous binder in the form of an adhesive ductile film coating the aggregates contributes to durability, stability and flexibility of a road structure. These properties are dependent on the relative film thickness, the properties of the bitumen, and the amount of compaction during construction. Temperature and rate of loading affect the strength of an asphaltic material; age hardening of asphalt affects the durability of the pavement.

FORMULAE

$$1 \quad \Delta = \frac{1.5 pr}{E} = \frac{1}{\left[1 + \left(\frac{z}{r}\right)^2\right]^{\frac{3}{2}}} \text{ at } \mu = 0.5$$

Where Δ = deflection
 p = unit load on the circular plate
 r = radius of the plate
 E = modulus of elasticity
 z = depth from surface
 μ = Poisson's ratio

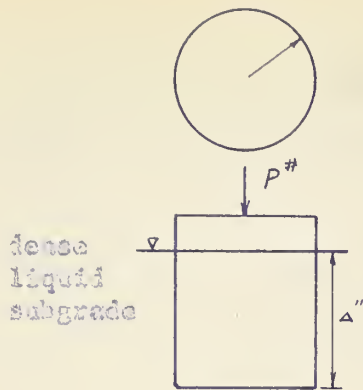
$$2 \quad Z_0 = \frac{P}{8kl^2} \left\{ 1 + [0.3665 \log_{10} \left(\frac{a}{l}\right) - 0.2174] \left(\frac{a}{l}\right)^2 \right\} \text{ equation (11) of reference 9}$$

Where Z_0 = deflection of the slab at the center of the load
 P = total pressure
 k = modulus of subgrade reaction
 a = radius of a circle over the area of which P is assumed to be distributed uniformly
 l = radius of relative stiffness

$$3 \quad \Delta = \frac{1.5 pr}{E_2} F_w ; F_w = f\left(\frac{r}{h}, \frac{E_2}{E_1}\right)$$

Where Δ = deflection
 p_1 = unit load on circular plate
 r = radius of plate
 E_2 = modulus of elasticity of lower layer
 F_w = dimensionless factor depending on the ratio of moduli of elasticity of the subgrade and pavement as well as the depth to radius ratio

4



floating
cylinder
loaded with $P^{\#}$

$$k = \frac{P}{\Delta} = \frac{P}{\pi r^2 \Delta} = \frac{\text{weight of displaced liquid}}{\text{volume of displaced liquid}}$$

= density of the liquid, pci

5

$$EI \frac{d^4 y}{dx^4} = -ky + q \quad (\text{differential equation for the deflection in curve of a beam supported on an elastic medium})$$

6

$$l = \frac{L}{\frac{Eh^3}{12(1-\mu^2)k}}$$

where l = radius of relative stiffness, inches
 E = modulus of elasticity of the pavement, psi
 h = thickness of the pavement, inches
 μ = Poisson's ratio of the pavement
 k = modulus of subgrade reaction, pci

7

$$k = \frac{P}{\Delta} = \frac{E}{2(1-\mu^2)r}$$

8

$$\Delta = \frac{2\pi r(1-\mu^2)}{E}$$

$$\frac{P}{\Delta} = k = \frac{E}{2r(1-\mu^2)}$$

$$k = \frac{E}{1.5r} \quad \text{at } \mu = 0.5$$

CHAPTER IV

THEORETICAL ANALYSIS OF EFFECTS OF VARIABLES

AFFECTING DEFLECTION BASED ON BURMISTER'S THEORY

To guide to interpret and to correlate the Penkelman beam data presented in the following chapter, theoretical analysis of the effects of variables affecting deflection in the pavement is carried out in the following sections.

THICKNESS AND STIFFNESS RATIO OF THE LAYERS

Based on Burmister's analysis, deflection, as stated, is influenced by the pressure intensity, the radius of contact area, the subgrade modulus, the stiffness ratio of the layered system, and the degree of concentration of the applied load which is expressed in terms of loaded area radius-pavement thickness ratio. Because the tire pressure is commonly assumed without significant error to be distributed over a circular area having a radius equal to that of an equivalent circular area of contact (63), the first two of the above-mentioned factors are constants depending on deflection-measurement procedure. The remaining variables determining the deflection are the stiffness ratio of the layer, the strength of the subgrade, and the thickness of the road structure. Stiffness can be achieved by employing materials having good strength properties or by thickening the layers. The former is indicated by the large magnitudes of the moduli of subgrade and the reinforcing layer materials. The modulus value, defined as the ratio of the unit stress to the corresponding unit strain for any value of stress below the proportional elastic limit (63), are determined by plate bearing

tests on top of the appropriate layers (20, 56, 63). In the evaluation of the moduli, Burmister considers the combination of the surfacing and the base as the top layer and the subgrade as the bottom layer in his two layered system (56). The unit load supported on a 30 inch diameter plate causing a total deflection of 0.2 inch is suggested for the moduli evaluation (20, 56, 63). The equation shown in Figure 2 with the exception that the constant is equal to 1.18 for the rigid bearing plate is used. For the computation of the modulus of elasticity of the subgrade, E_2 , the settlement coefficient, F_w in the equation is equal to unity because the test is carried out on the subgrade. For the determination of the modulus of elasticity of the top layer, E_1 , the plate bearing test on the layer of known thickness is made in the same manner as that on the subgrade, and the settlement coefficient is computed from the equation. After obtaining the value of the coefficient, the modulus ratio of the materials is found from the influence curve shown in Figure 2, and thus the value of E_1 is computed (20, 38, 63).

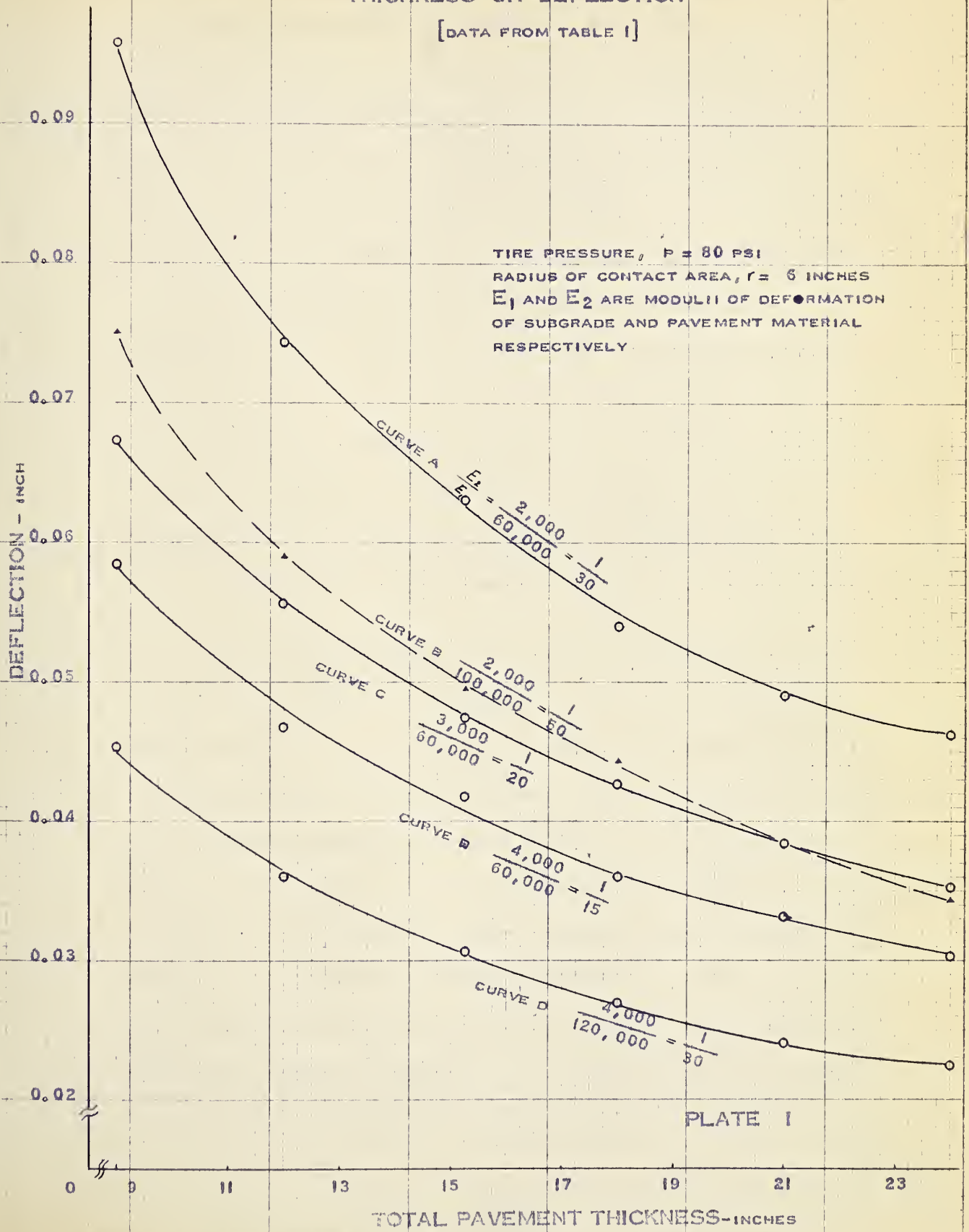
The suggested limiting deflection of 0.2 inch is only an arbitrary value and it might have to be changed in the light of experience as pointed out by Burmister (56). In the evaluation of the WASHO Road test (62), Burmister indicates that the evaluation of the moduli of the pavement materials requires

pressure-deflection data on different sizes of bearing plates and the data should be representative of and comparable to the probable actual imposed stresses in service of the pavement layers. Therefore it seems that the number of load repetition, the magnitude of unit load and the limiting deflection under a bearing plate for the evaluation of modulus values of a pavement are dependent on the characteristics of the pavement materials.

As shown in Plate 1, the assumed values of the modulus of deformation of the subgrade range from 2,000 to 4,000 psi, which correspond approximately to moduli of subgrade reaction of 113 to 226 pci from $k = \frac{E_2}{11.8r}$ (56, 70) or to California Bearing Ratio of 3.4 to 14.5 per cent from the curve showing the relationship between CBR and k by Middebrooke and Bertram (17) respectively. The values represent relatively weak to strong subgrade soils of inorganic clays of low to medium plasticity (35), which are the predominant subgrade material underlying the pavements studied in this dissertation. The plots, which are in accordance with the basic two layer settlement equation and with the influence curves as shown in Figure 2, indicate the effect of the moduli and the pavement thicknesses on deflection. The deflection decreases as the thickness of the pavement increases, and the rate of decrease is influenced by the stiffness ratio, the stiffness of the subgrade, and the thickness of the structures. With relatively thin pavements, regardless of the moduli, the effect of increas-

EFFECT OF MODULUS OF DEFORMATION OF PAVEMENT MATERIAL AND PAVEMENT THICKNESS ON DEFLECTION

[DATA FROM TABLE I]



ing pavement thicknesses on deflection is pronounced, as reflected in a rapid reduction in deflection values; however, the effect of the thickening becomes less pronounced as the thickness of the layer is increased. Curve A indicates that increasing the thickness from 9 inches to 10 inches results in a reduction of deflection from 0.0958 inch to approximately 0.0886 inch, and that increasing from 23 inches to 24 inches results in a reduction of deflection from 0.0468 inch to 0.0461 inch, about 8 per cent versus 1.6 per cent decrease in deflection. Because of the influence of different moduli of the pavement materials, the relative effect of thickening on deflection is different, as shown by the different slopes of the deflection curves. It is to be noted that curve B intersects curve C.

At a given thickness, the change in stiffness ratio due to a change in modulus of the pavement or the subgrade affects the deflection. Curves A, B, and C show that the deflection of a pavement having a 15-inch thickness decreases from 0.0630 inch to 0.0496 inch and 0.0475 inch as a result of increasing the modulus of the pavement surface from 60,000 to 100,000 psi and of increasing the subgrade modulus from 2,000 to 3,000 psi respectively. Changing both the subgrade and the pavement moduli but maintaining a constant stiffness ratio will also affect the deflection, which is shown by curves A and D. Thus, for example, the deflection of a 15-inch pavement is found to decrease from

0.063 inch to 0.0306 inch (a reduction of 51.4 per cent) when the individual moduli of the subgrade and of the pavement are doubled at unaltered value of the stiffness ratio. The family of curves in Plate 2 gives a clearer picture concerning the above explanation. For a constant pavement thickness of 15 inches and at a constant stiffness ratio, increasing the subgrade modulus results in decrease in deflection, but the effect of increasing the subgrade modulus is decreasing. Increasing the stiffness ratio, on the other hand, also results in decrease in deflection, and this effect is much pronounced with relatively thin pavements. It is noted by comparison of the change in slopes of the curves at stiffness ratios of from 5 to 10, and from 95 to 100.

For the purpose of further analysis of the effect of change in absolute values of bearing moduli and their ratios with respect to the required pavement thickness based on a limiting deflection criterion (1, 18, 24, 48), Plate 3 has been constructed. The assumed deflection value of 0.05 inch is only an arbitrary value for the purpose of this analysis. It is seen that, at constant E_1 or E_2 a change in their ratio results in changing the required pavement thickness, and that at a constant ratio, change in the moduli also affects the thickness. To meet the design criterion of limiting deflection, the curves indicate that there are two choices in stiffening the pavement: to employ materials having high strength properties or to thicken

EFFECT OF STIFFNESS OF PAVEMENT ON DEFLECTION FOR A 15 - INCH PAVEMENT [DATA FROM TABLE 2]

$P = 80 \text{ PSI}$
 $r = 6 \text{ INCHES}$

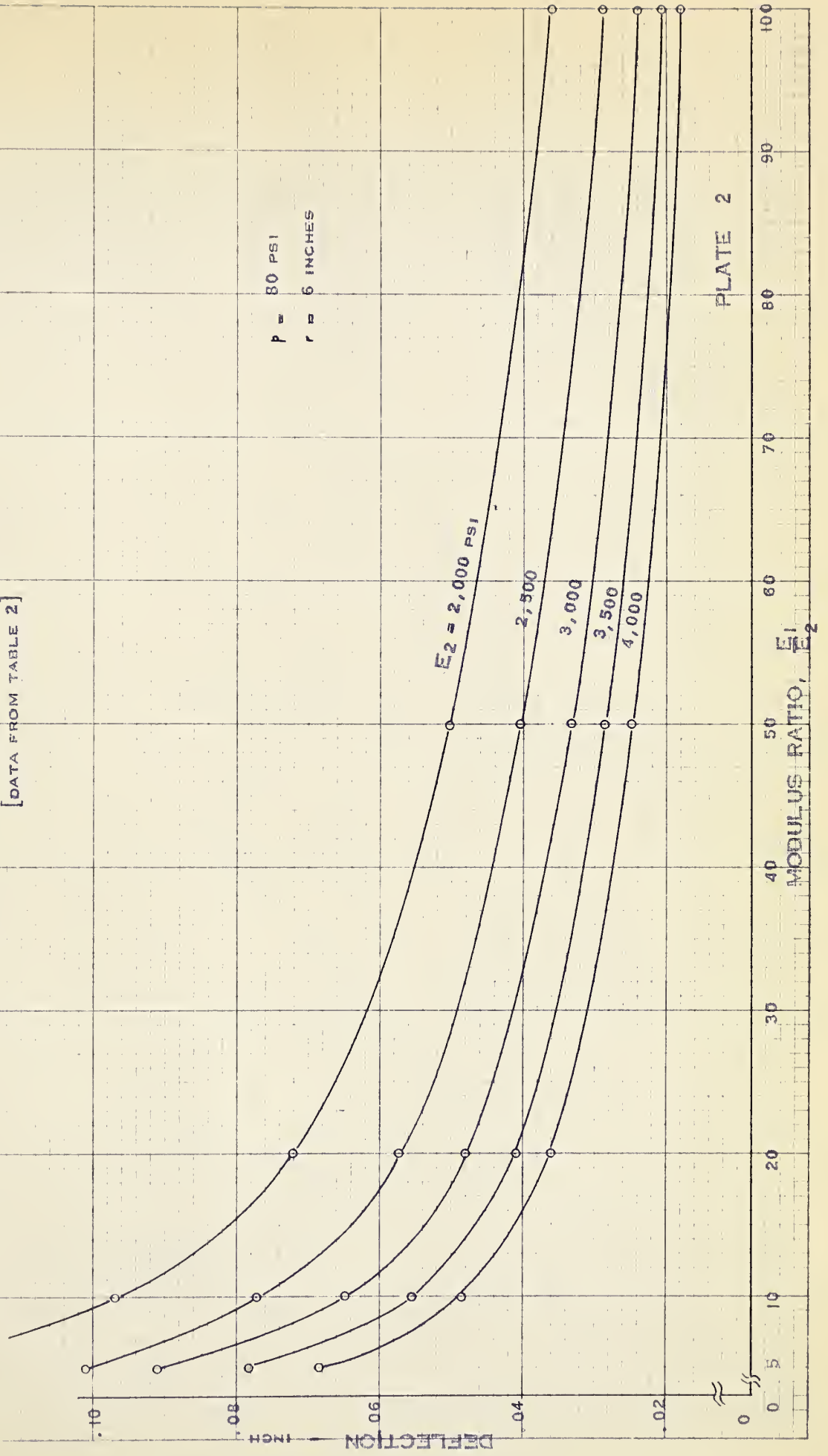


PLATE 2

0 0.02 0.04 0.06 0.08 0.10

MODULUS RATIO, E_1/E_2

DEFLECTION INCHES

EFFECT OF $\frac{E_2}{E_1}$ AND E_2 ON PAVEMENT
THICKNESS REQUIRED FOR AN ARBITRARY
LIMITING DEFLECTION OF 0.05 INCH
[DATA FROM TABLE 3]

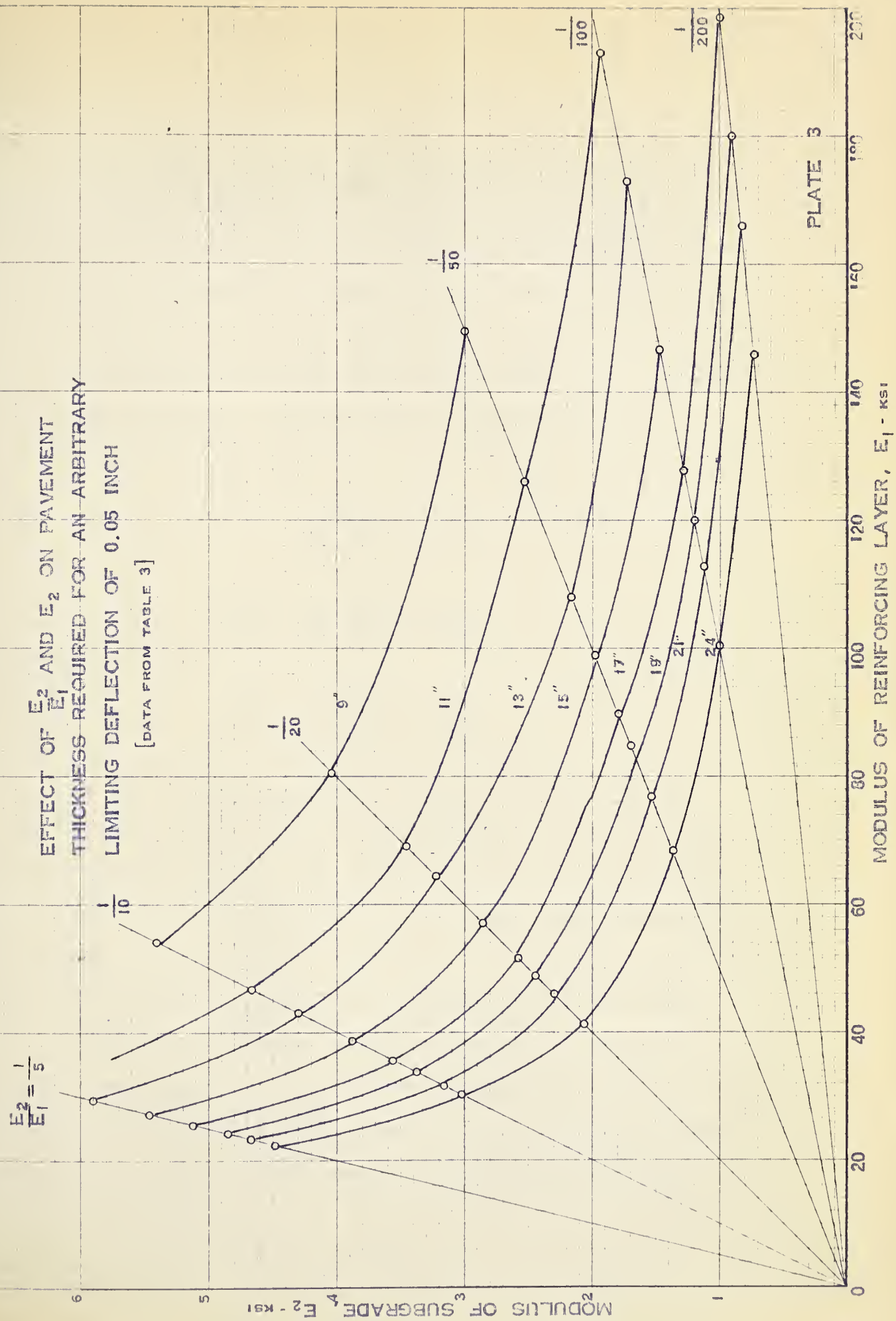


PLATE 3

the pavement. An attempt has been made to set up empirical equations to express the relationship of the stiffness ratios and the required pavement thicknesses based on constant subgrade strengths and an assumed limiting deflection value. In doing so, moduli in the range of the strength of clays of low to medium plasticity have been adopted: 2,000, 3,000, and 4,000 psi (17, 56, 70, 35). The result of the analysis based on method of least squares* gives the following empirical equations:

$$\frac{E_1}{E_2} = \frac{1}{0.00271 h - 0.0197} \quad (A)$$

$$\frac{E_1}{E_2} = \frac{1}{0.00546 h - 0.0278} \quad (B)$$

$$\frac{E_1}{E_2} = \frac{1}{0.00798 h - 0.0202} \quad (C)$$

in which 'h' denotes the total pavement thickness. The above equations are based on the assumed subgrade moduli of 2,000, 3,000, and 4,000 psi respectively. Equation (A) is valid for pavement thicknesses ranging from 13 to 25 inches, and for the strength property of the combination of the asphaltic surface and the base of approximately 20 to 60 times that of the subgrade assumed. Equations (B) and (C) are valid for pavement thicknesses ranging from 9 to 24 inches and for the strength property of the combination of the asphaltic surface and the base of 10 to 50 times and 6 to 20 times that of the subgrades assumed

* see Appendix IV

respectively. The above moduli for the combination of the surfacing and the base so determined are found to be from 40,000 to 120,000 psi; from 30,000 to 150,000 psi; and from 24,000 to 80,000 psi for the assumed subgrade moduli of 2,000, 3,000, and 4,000 respectively in the pavement thickness ranges stated. Not all the above values are found to fall within the modulus range obtained by Burmister on the WASHO Test road, which range from 40,000 to 160,000 psi. This is because the field conditions from which the moduli have been obtained are different from the assumed conditions based on which the empirical equations are derived. The combined thickness of the surfacing and base in the WASHO test road is underlain by various thicknesses of sub-base having an average modulus value of 17,100 psi, and the sub-base, in turn, is underlain by a 24-inch compacted subgrade having an average subgrade modulus of 6,400 psi. The use of an arbitrary deflection of 0.05 inch in the derivation of the equations also affects the values of moduli computed from the equations. It is seen from the equations and from the curves in Plate 4 that large reductions in thicknesses are permissible when employing materials having better strength properties, assuming the compaction is adequate in each case.

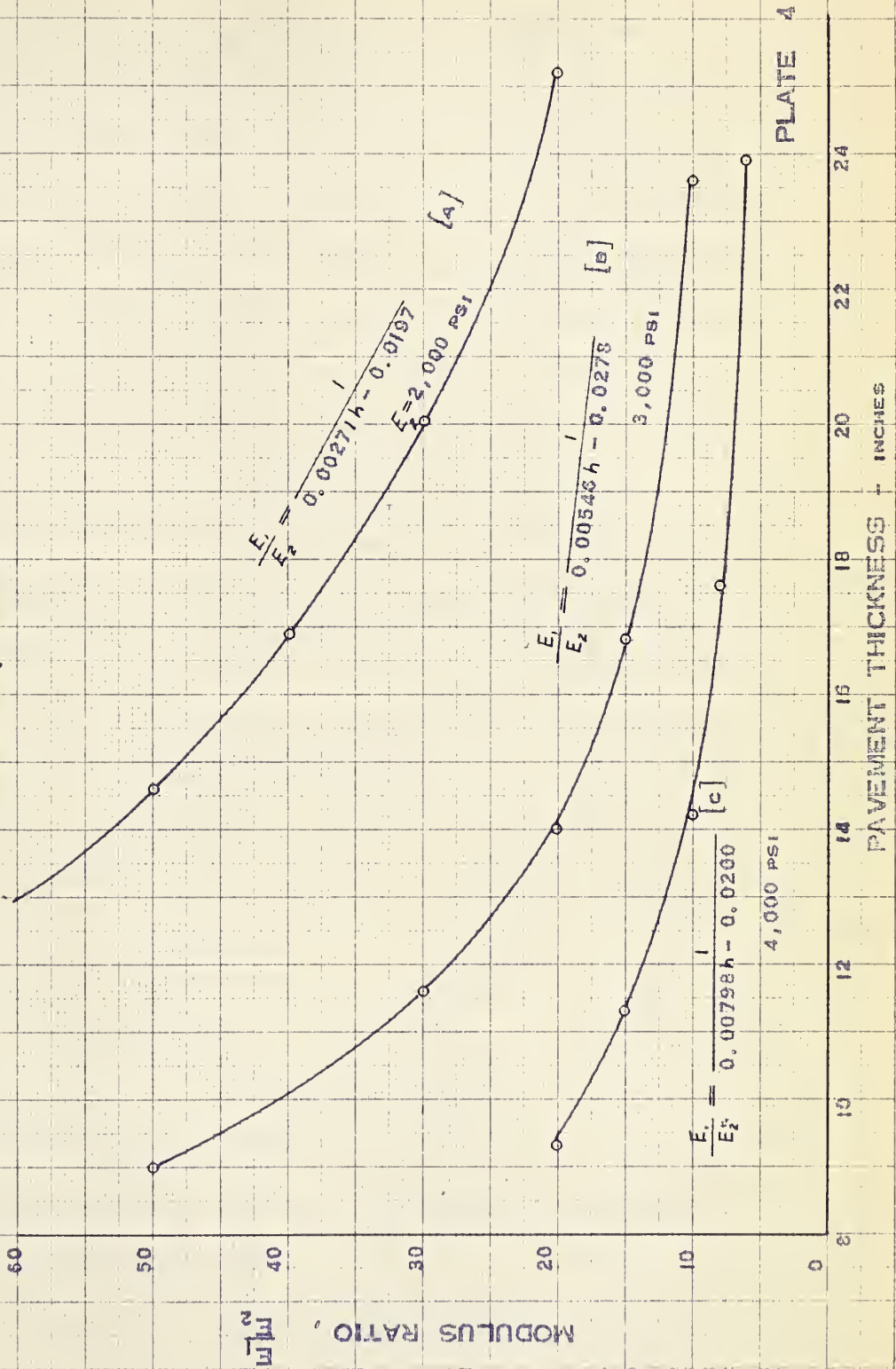
Temperature

As asphaltic material is thermoplastic, its modulus of de-

$\frac{E_1}{E_2}$ vs PAVEMENT THICKNESS REQUIRED FOR A LIMITING DEFLECTION OF 0.05 INCH AND FOR VARIOUS SUBGRADE MODULI

MODULI

[DATA FROM TABLE 4]



formation is dependent on temperature at a constant time of loading, and, consequently the change in temperature affects the deflection of a pavement. To analyze this effect, Plates 5 to 9 inclusive were made. In the preparation of these plates, various subgrade moduli were assumed and the moduli of deformation of asphaltic materials at various temperatures obtained by Nijboer (41) and Papazian and Baker (67) were utilized. Because of difference in test procedures on different asphaltic materials, the moduli of deformation so obtained by the investigators* were different. However, this indicates that the strength evaluation of material is influenced by the test procedures and the interpretation of the results. In this analysis, the modulus values obtained by each investigator are treated separately, and the result of the analysis is considered to be on a qualitative rather than quantitative basis in showing the effect of temperature on deflection due to change in modulus values as temperature changes.

* Ranges of moduli of asphaltic materials:

Nijboer	14,000 to 280,000 psi
---------	-----------------------

Papazian and Baker	3,400 to 18,000 psi
--------------------	---------------------

Ranges of moduli for the combination of surfacing and base obtained by Burmister on WASHO test road:

40,000 to 160,000 psi

EFFECT OF CHANGE IN MODULUS OF
DEFORMATION OF BITUMINOUS MATERIAL
DUE TO TEMPERATURE CHANGES ON
SURFACE DEFLECTION - I

[DATA FROM TABLE 5]

E_1 VALUES ARE OBTAINED FROM
[NIJBOER'S DATA ON TEMPERATURE-
MODULUS RELATIONSHIP]

SURFACE DEFLECTION - $\times 10^{-2}$ INCH

CURVE A $\frac{E_2}{E_1} = \frac{4,000}{14,000}$ AT 68°F

CURVE B $\frac{4,000}{70,000}$ AT 41°F

CURVE C $\frac{4,000}{280,000}$ AT 14°F

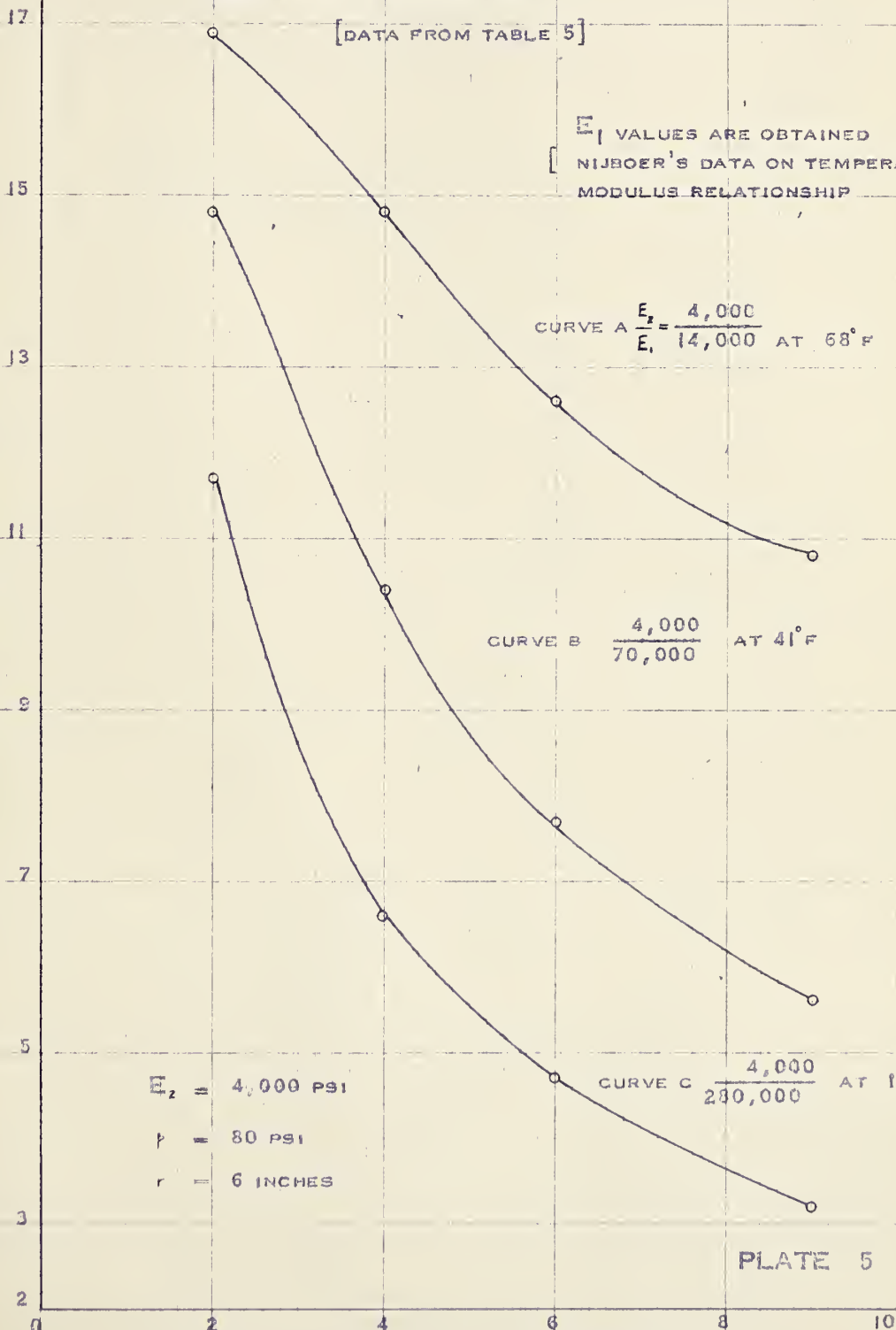
$E_2 = 4,000$ PSI

$p = 80$ PSI

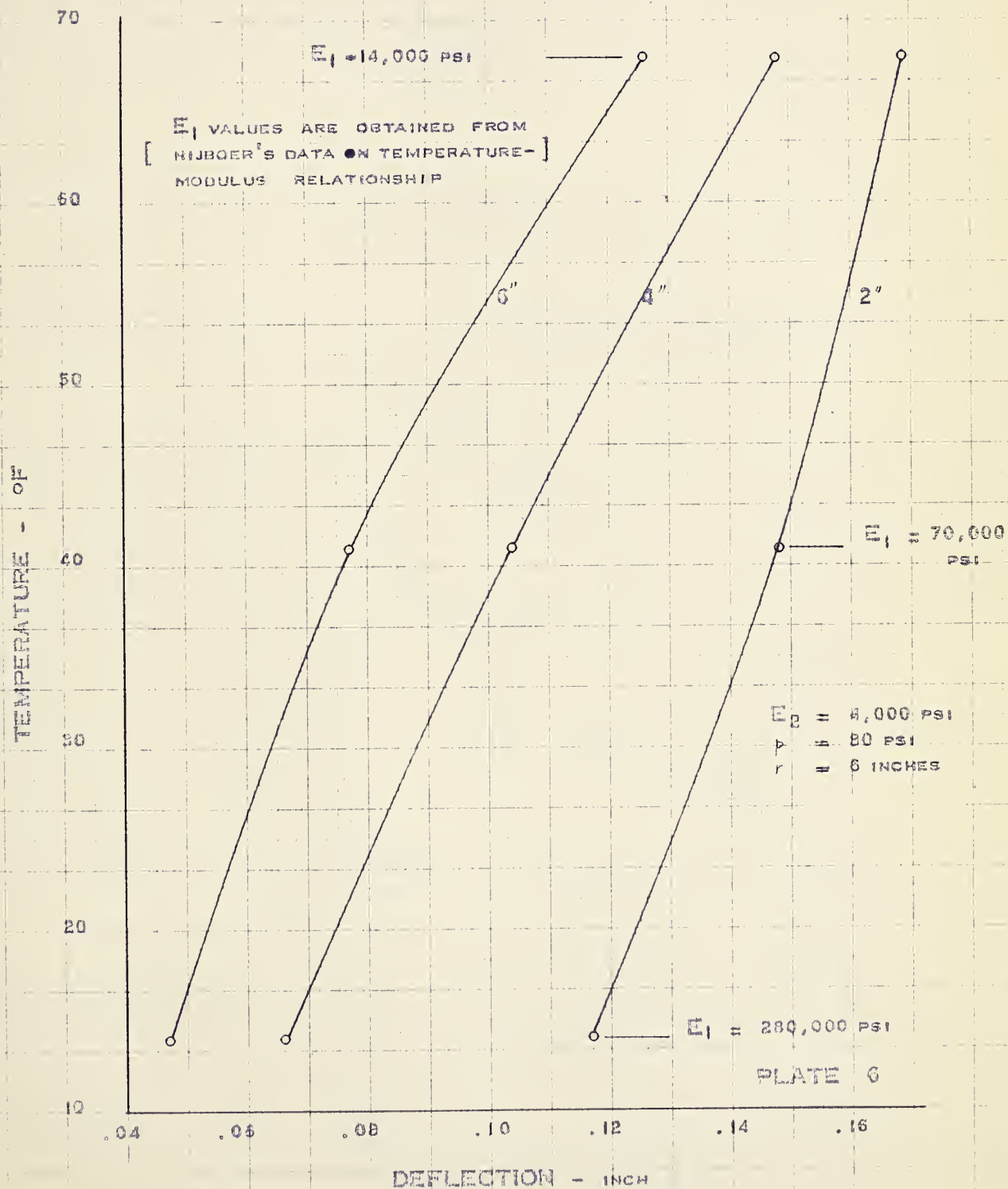
$r = 6$ INCHES

PLATE 5

THICKNESS OF BITUMINOUS SURFACE - INCHES



EFFECT OF CHANGE IN TEMPERATURE ON SURFACE
DEFLECTION FOR VARIOUS SURFACE THICKNESSES - 1
[DATA FROM TABLE 5]



EFFECT OF CHANGE IN MODULUS OF DEFORMATION OF BITUMINOUS MATERIAL SURFACE DEFLECTION - 2

[DATA FROM TABLE 6]

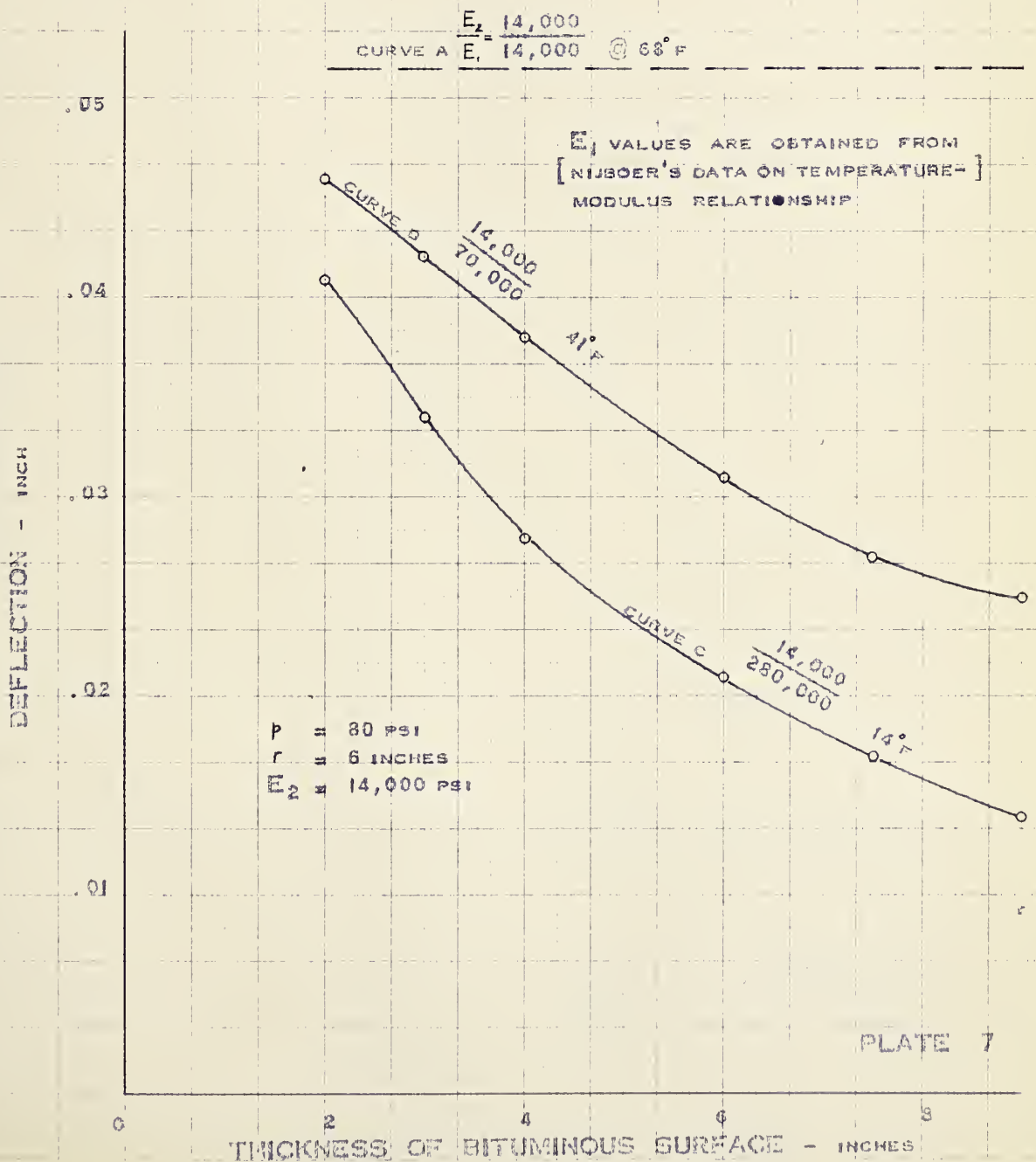
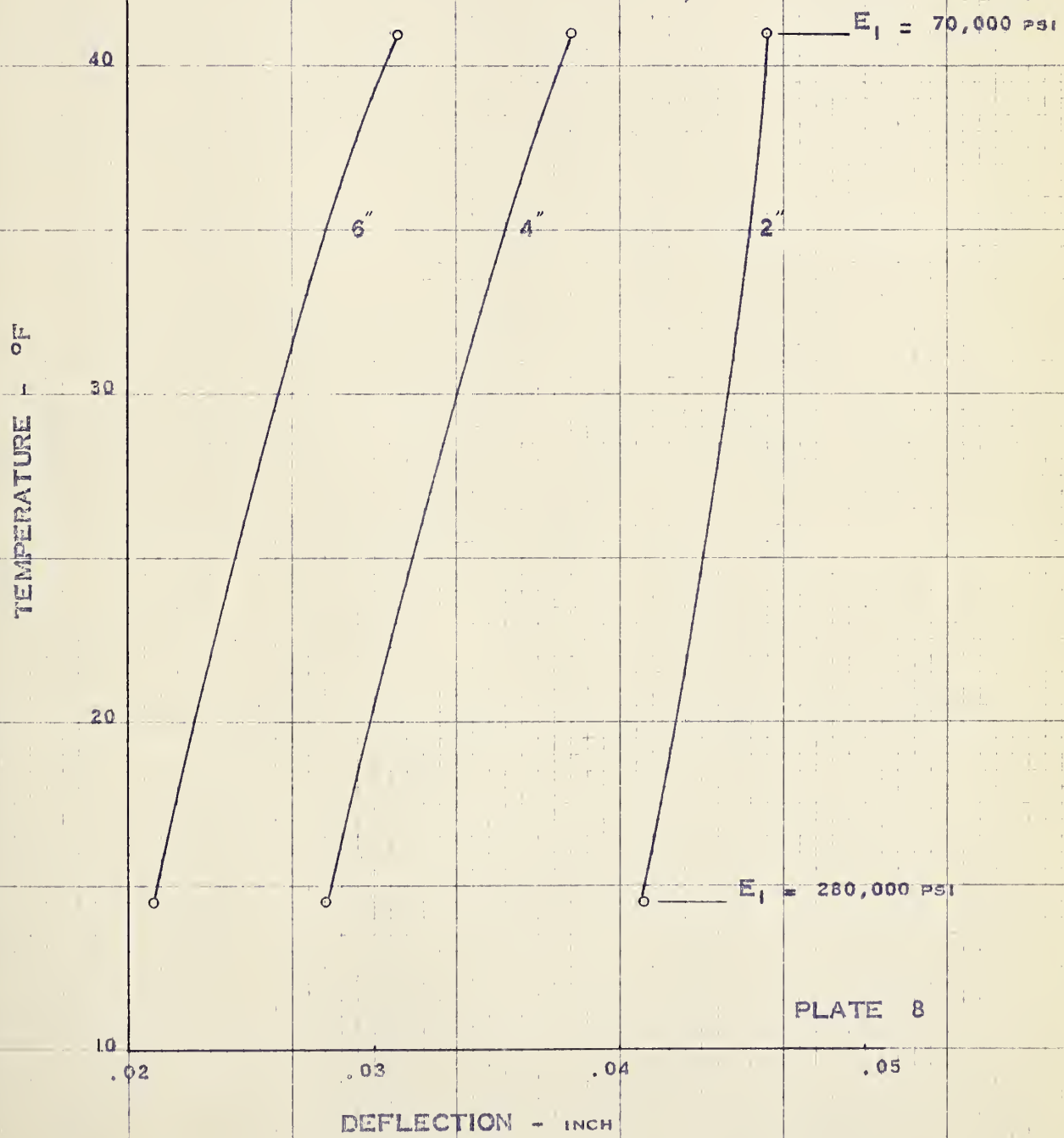


PLATE 7

EFFECT OF CHANGE IN TEMPERATURE ON SURFACE DEFLECTION FOR VARIOUS SURFACE THICKNESSES - 2

[DATA FROM TABLE 6]

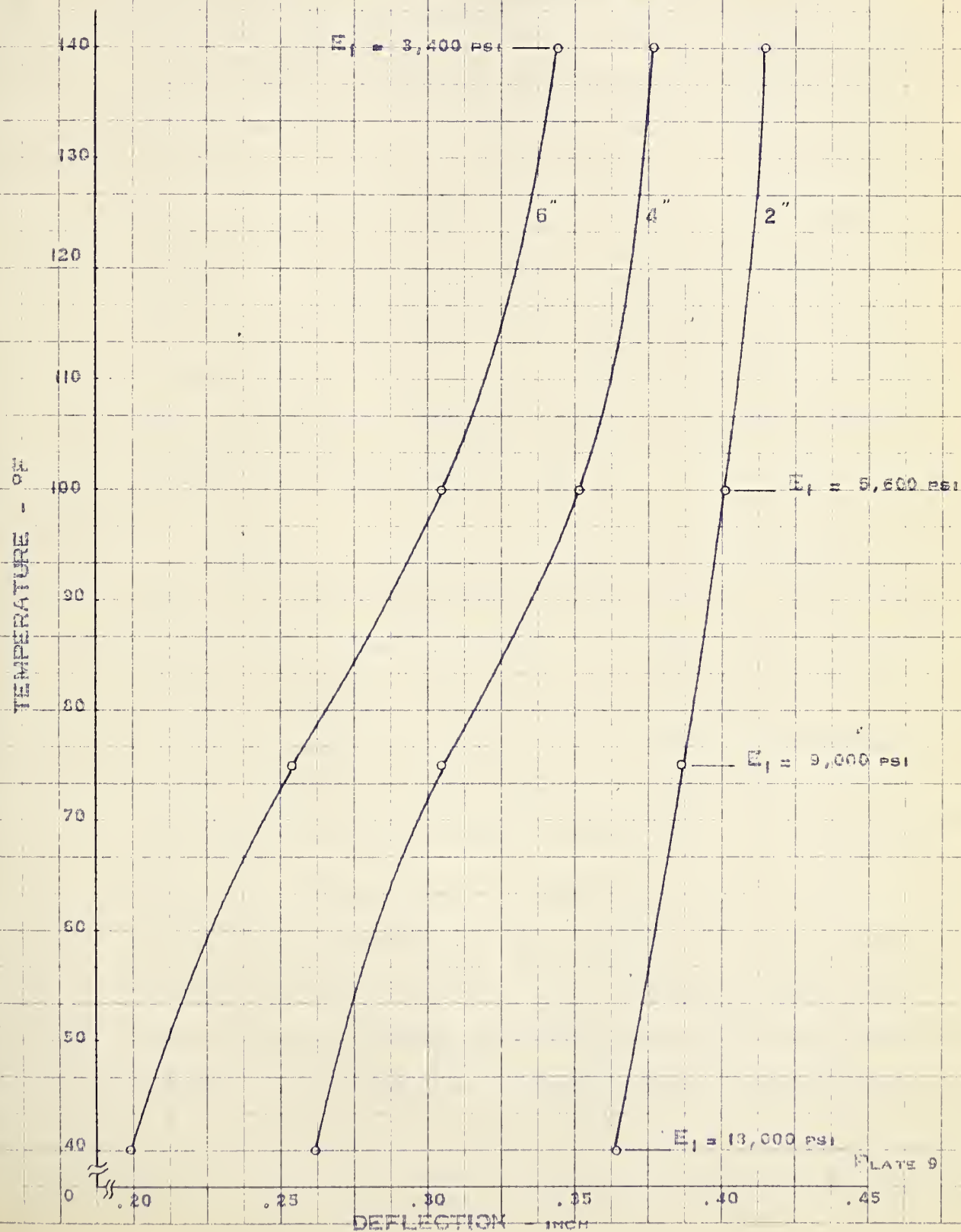
[E_1 VALUES ARE OBTAINED FROM NIJBOER'S DATA ON TEMPERATURE-MODULUS RELATIONSHIP]



EFFECT OF CHANGE IN TEMPERATURE ON SURFACE DEFLECTION FOR VARIOUS SURFACE THICKNESSES - 3

[DATE FROM TABLE 7]

[E_1 VALUES ARE OBTAINED FROM BAKER'S DATA]
ON TEMPERATURE-MODULUS RELATIONSHIP



Plates 5 and 7 show the change in deflections brought about by changes in the moduli of deformation of the asphaltic materials of various thicknesses as a result of change in temperatures with a constant subgrade modulus which is considered independent of surface temperature. The computation of the deflection is based on Burmister's deflection equation noted. It is seen that the deflection increases as the temperature increases for a given thickness of the material. The vertical distances between two curves at various pavement thicknesses indicate that the effect of temperature on deflection is more pronounced in the pavement having thick asphaltic surfacing. This is in agreement with the findings of the WASHO road test (46).

The rate of increase in deflection due to increase in temperature is not only dependent on the thickness of the surface but also on the temperature range. The relative rate of increase in deflection within a temperature ranges of 14 to 68° F and 40 to 140° F is shown by the slopes of the temperature-deflection curves in Plates 6, 8, and 9. From the curves in Plate 9, it is seen that the relative effect of temperature on deflection is decreasing as the temperature is increasing. For the 2-inch bituminous surface, increasing the temperature from 40 to 75° F causes an increase in deflection from 0.364 inch to 0.386 inch, while increasing the temperature from 105 to 140° F, the increase in deflection is approximately from 0.405 inch to 0.415 inch (or

6.1 per cent against 2.5 per cent). Based on the curves in Plate 9, the effect of temperature on deflection seems pronounced up to approximately 100° F for the 2- and 4- inch bituminous surfaces, which are the common thicknesses employed in highway pavements.

From the curves in Plates 6 and 9, it is seen that the temperature-deflection curves based on the modulus-temperature relationships furnished by Nijboer (41) are not identical with those based on similar data furnished by Papazian and Baker (67). With a temperature change from 41 to 68° F, the increase in deflection for the 2-inch asphaltic material is 14.4 per cent in the former against 7.1 per cent in the latter, and for the 4-inch material, 41.4 versus 15.4 per cent. The corresponding magnitudes of deflection as listed in Tables 5 and 7 are also not comparable. The reason may be attributed to the different procedures of compression test used by the investigators and to the different rates of loading employed. A time of loading of 100 seconds and a rate of deformation of 0.005 inch per minute were taken from Nijboer and Papazian and Baker's E-rate of loading curves respectively. No correlation between these two different rates of loading can be made so as to compare the modulus values on the same basis. The modulus values so adopted for the analysis are yet, the best available data at hand.

Tire Pressure and Contact Area

Deflection is a function of both tire pressure and contact area. At a constant load, the contact area varies inversely with the tire pressure. The effect of increase in contact area due to lower tire pressures results in a greater radius of a deflection curve in a pavement; however, this effect is reported (23) to have little significance for tire pressures ranging from 50 to 90 psi.

In the CGRA deflection-measurement procedure, standard vehicle load and tire pressure are specified, and thus the contact area is constant. The effect of variation of tire pressure on deflection as shown in Plate 10 is therefore only for the purpose of analysis, because large variation in tire pressure during the deflection measurement is not anticipated. The curves in the plate show that a variation of ± 5 psi from the standard tire pressure of 80 psi causes little change in deflection. For a 15-inch pavement, such a variation causes only a change in deflection by about 2 per cent. Approximately the same variation is found for other pavements thicknesses.

EFFECT OF TIRE PRESSURE ON DEFLECTION

[DATA FROM TABLE 8]

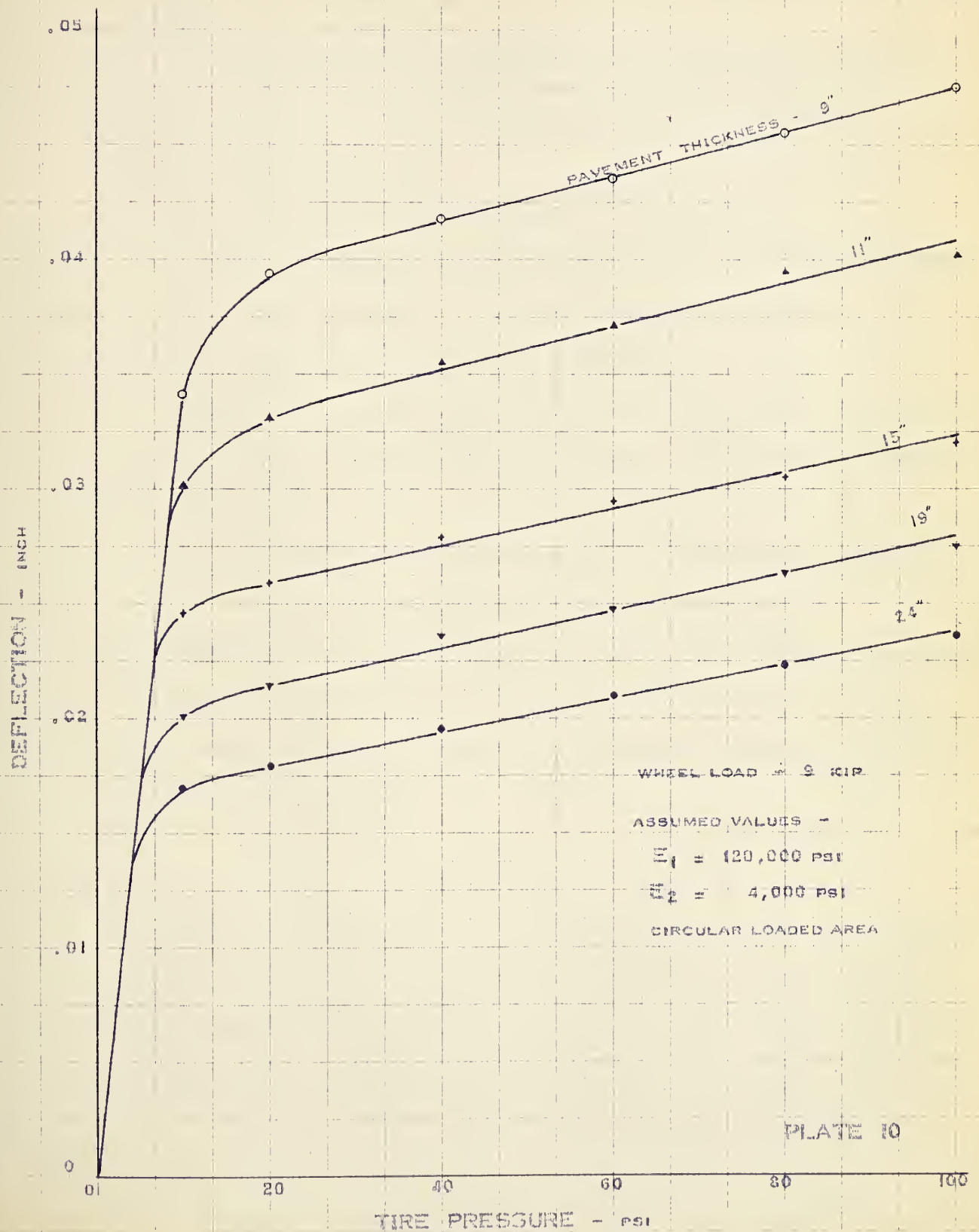


PLATE 10

CONCLUSIONS

From the theoretical analysis, the following conclusions may be drawn:

1. Surface deflection is influenced by the thickness of a pavement and the relative stiffness of the materials in the component parts at constant traffic conditions.

2. It is possible to reduce pavement deflections by:

- (i) increasing the degree of compaction of the component parts of the pavement within a certain range to assure a full continuity in the layer interfaces;
- (ii) using materials with superior load-distribution properties to increase the relative stiffness of the pavement component; and/or
- (iii) increasing the over-all pavement thickness to decrease the effect of concentration of the applied load.

3. Based on an arbitrary allowable deflection value of 0.05 inch and on different subgrade moduli, the relationship between the stiffness ratio and the pavement thickness required may be expressed by empirical equations of the form:

$$\frac{E_1}{E_2} = \frac{1}{A h - B}$$

The values of the constants in the equation are dependent on the assumed subgrade moduli which constitute the applicability of the equation for various pavement thickness and modulus ratio ranges. Most of the moduli so computed by the equations fall within the modulus range obtained by

Burmister on the WASHO test road.

4. Based on the limited information, deflection is found to increase as temperature increases, and the effect of temperature on deflection is found to be more pronounced on pavements having thick asphaltic surfaces. An increase of temperature from 40 to 100° F results an increase in deflection of 10.4 per cent and 34.3 per cent for the 2- and the 4-inch bituminous surface respectively, and the effect of temperature on deflection seems to be pronounced up to a temperature of approximately 100° F.

5. The effect of a tire pressure change of ± 5 psi from a standard tire pressure of 80 psi causes only about 2 per cent variation in deflection for the assumed moduli of 4,000 and 120,000 psi of the subgrade and the reinforcing layer material respectively.

CHAPTER V

INTERPRETATION AND CORRELATION OF BENKELMAN BEAM DATA

Pavement deflection measurements on sections of various highways in the Province of Alberta were made by the Alberta Highways Department in accordance with the pavement investigation program specified by the Special Committee on Pavement Design and Evaluation of the Canadian Good Roads Association (65, 69). Measurements were taken by means of Benkelman beam at fixed points or at random intervals within a homogeneous section on the outer wheel path of the roadways. A 9-kip wheel load equipped with 11.00 x 20 dual tires inflated to 80 psi was used for the load-deflection tests. The four highways considered in this investigation are Alberta Highways 9, 13, 14 and 16 because extensive Benkelman beam data had been taken in the various sections of the highways. The sections of the highways vary in length, subgrade soil types, and in geometric features. There are also differences in traffic coverages and chronological ages of the pavements. The relative locations of the highways are shown in the map in Appendix II. For clarity, the information regarding the four highways is summarized in Table 9 to 12.

TABLE 9

Summary of Information Regarding the
Pavement Sections Studied - I

Highway #9

Section No.	Length Mi.	Total pav't thickness in.	Subgrade Soil Type	Unified Soil Classification System	CGRA Code	Chronological age of pav't	Traffic Coverage Heavy Axle Coverages X 100 lane	Shoulder Type	Con-struction
1 - 6	28.3	2.5+6=8.5	CL		10	7	2170-2310	7	Full depth granular courses, 3 to 6 feet wide, unpaved surface.
7 - 8	6.1	2.5+9=11.5	CL		10	6	2290	9	
9	4.4	2.5+9=11.5	CL-CH		10*	6	1530	9	
10 - 15	19.0	2.5+9=11.5	CH		13	6	1530	9	Full depth granular courses, 6 feet wide, paved surface.
16	0.9	4+9=13	CL		10	5	1270	9	
17 - 22	14.0	4+11=15	CH		13	4	1020-1345	9	
23 - 25	1.6	4+11=15	SP		6	4	1163	9	
26 - 30	7.3	4+11=15	CL		10	4	1163	9	
31 - 33	5.3	5+9=14	CL-CH		10*	4	1163	9	
34	1.8	4+9=13	CL		10	4	1163	9	
35	0.2	4+9=13	CL-CH		10*	4	1163	9	

TABLE 10 - II

Highway #13

1 - 12	19.7	2.5+6=8.5	CL			7	2292	8	Full depth granular courses, 0 to 3 feet wide, unpaved surface.
13	5.4	2.5+6=8.5	CL			7	2842	8	
14 - 16	26.0	2.5+9=11.5	CL			6	1873	9	Full depth granular courses, over 6 feet wide, paved surface.
17 - 20	18.2	3+9=12	CL			5	1623	5	
21 - 23	4.7	4+9=13	CL			2	341	5	
24	9.6	4+9=13	SP			2	341	5	
25 - 27	6.4	4+9=13	CL			2	174	5	
28 - 29	4.2	4+9=13	SP			2	174	5	

TABLE 11 - III

Highway #14

Section No.	Length Mi.	Total pav't thickness in.	Subgrade Soil Type	Chronological age of pav't	Traffic Coverage Heavy Axle Coverages X 100 lane	Shoulder Type	
			Unified Soil Classification System				CGRA Code
1	0.9	4.5+9=13.5	CH	6	816	5	
2	0.7	2.5+6=8.5	CH	6	816	6	
3 - 13	18.5	2.5+6=8.5	CL	7	1893	6	Full depth granular courses, over 6 feet wide, unpaved surface.
14 - 17	13.5	2.5+9=11.5	CL	6	1623	9	
18 - 20	6.0	3+9=12	CL	3	868	9	
21	1.5	3+9=12	CL	3	868	5	
22 - 23	6.5	3+9=12	CL	3	1096	5	
24 - 25	1.4	3+12=15	CL	3	1096	5	
26 - 27	9.5	4+9=13	CL	3	1096	5	
28 - 29	16.2	4+9=13	CL	2	774	5	
30	6.8	4+12=15	CL	2	774	5	
31	2.9	4+14=18	CL	2	774	5	
32	8.5	4+14=18	CL	1	395	5	
33 - 37	13.3	4+12=16	CL	1	395	5	

TABLE 12 - IV

Highway #16

4 - 22	24.7	4.5+9=13.5	CH	10	9622	3	Trenched construction, 3 to 6 feet wide, unpaved surface.
23 - 29	15.1	4+10=14	CL	8*	4350	9	
30 - 34	11.7	4+10=14	CL	8*	2920	9	
35 - 36	11.4	3+11=14	CL	4	2920	5	
37 - 40	12.5	3+11=14	CL	4	2435	5	
41 - 42	4.0	3+9=12	CL	5	3040	5	
43 - 45	24.6	3+9=12	SC	5	3040	5	
46	2.3	3+9=12	CL	5	3040	5	
47 - 49	11.3	5.5+3=8.5	CL	7	4255	4	
50 - 52	8.3	5.5+3=8.5	CL	7	3310	4	
53 - 59	12.9	5.5+3=8.5	CL	8	3600	4	Trenched construction, 0 to 3 feet wide, unpaved surface.
60 - 62	15.3	5.5+3=8.5	CL	10	4130	4	

* resurfaced: 4 years

DEFLECTIONS

Deflection Versus Thickness of Pavement

Plate 11 shows the average deflection values obtained from the four highways on sections having different pavement thicknesses. The figures beside the points denote the number of individual deflection measurements, from which the mean values were computed. The wide spread of points as shown may be attributed to the fact that the deflections were taken at various highways constructed under different field conditions and subjected to different traffic conditions. However, the deflections taken at the same section having the same geometric features and having the same traffic coverages were also found to vary considerably, as indicated by the plotted points representing one standard deviation on each side of the mean values in Plate 14. Such inconsistent deflection values have been previously reported (44,59).

For regression analysis on the relation of deflection and pavement thickness, the Benkelman beam deflection values taken on sections having performance ratings lower than 6* have been eliminated. Based on the data so treated, an empirical equation has been obtained by visual estimation as :

$$\Delta = \frac{0.47}{h} \quad (A)$$

and by an electronic computer as :

$$\begin{aligned} \Delta = & -1.5741 + (5.1600 \times 10^{-1})h - (5.9191 \times 10^{-2})h^2 \\ & + (2.9159 \times 10^{-3})h^3 - (0.5254 \times 10^{-4})h^4 \end{aligned} \quad (B)$$

* a rating of 6 is the boundary between fair and good performance based on the Special Committee on Pavement Design and Evaluation of CGRA(69).

DEFLECTION vs PAVEMENT THICKNESS FOR THE FOUR HIGHWAYS

[DATA FROM TABLE 13]

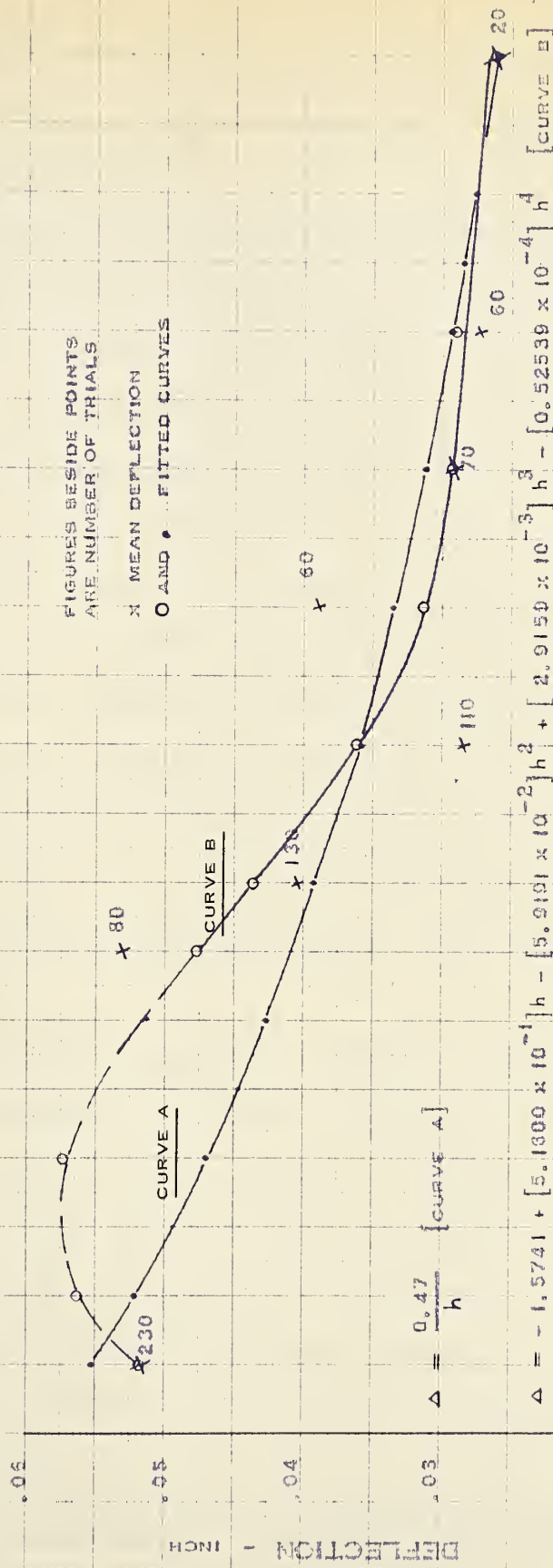


PLATE 11

In these equations, ' Δ ' denotes the deflection and 'h' denotes the total pavement thickness, both in inch units. The respective curves are plotted in Plate 11.

It is seen from the plate that the deflection decreases as the total pavement thickness increases as indicated in the theoretical analysis, except for sections having pavement thicknesses less than $11\frac{1}{2}$ inches which are shown by the dashed portion of curve (B). This disagreement between the field data and the result of the theoretical analysis leads to further examination of the field data. It is found that the deflection values for the sections having pavement thicknesses of $8\frac{1}{2}$ and $11\frac{1}{2}$ inches are the arithmetic means of 230 and 80 individual measurements, and in reducing the numbers of measurements to the basis of per unit length of the pavements, these deflection values are only the means of 4.8 and 1.9 measurements per mile at the two sections respectively. These average deflection values with high standard deviations* about their means would appear to have limited use as typical values.

It was thought desirable to correlate the best fitted curve (B) of the mean Benkelman beam data to a theoretical curve so as to determine how closely the theory studied approximates actual conditions. The construction of the theoretical curve necessitates comparable and consistent deflection data at different thicknesses of pavements, from which the moduli of the materials at the sections can be evaluated (20, 26). The method of evaluation involves a graphical trial

* The standard deviations are as high as 0.019 inch about the means of ten random measurements within a homogeneous section of the pavements.

solution, the procedures of which are as indicated in Table 15 and Plate 12. The values in column 4 of the table are the average deflections measured by Benkelman beam at the corresponding sections having various pavement thicknesses as listed in column 3. The settlement coefficients, F_w , in columns 7, 8 and 9 are computed from the basic settlement equation for the trial subgrade moduli, E_s . The trial solution is illustrated in Plate 12, which is the influence curve of the settlement coefficient as shown in Figure 2. From the graph of F_w versus $\frac{h}{r}$, it is seen that most compatible values in conformity with the pattern curves are 4,000 psi* for the subgrade modulus and 35 for the modulus ratio. Thus, the modulus of deformation of the reinforcing layer then equals to 140,000 psi*.

The above modulus values so determined are the strength properties of the pavement studied, and are not the average value of all pavements. With these moduli, a theoretical curve is plotted and is extended to a pavement thickness of 9 inches, as shown in Plate 13, so as to compare with the best fitted curve previously obtained. It is seen that the two curves almost coincide at pavement thicknesses from approximately 14½ to 18 inches. This indicates that the theory is adequate to explain the real phenomenon in the pavements, and thus the theoretical equation shown in the plate can be used for predicting the deflections for pavements having the thickness range and having the strength properties mentioned therein. However, at pavement thicknesses

* The moduli obtained by Burmister in his evaluation of the WASHO Road test (62) are: natural subgrade - 2,000 psi; 24-inch compacted subgrade - 5,800 to 7,000 psi; subbase - 12,000 to 26,000 psi; combined surfacing and base - 40,000 to 160,000 psi.

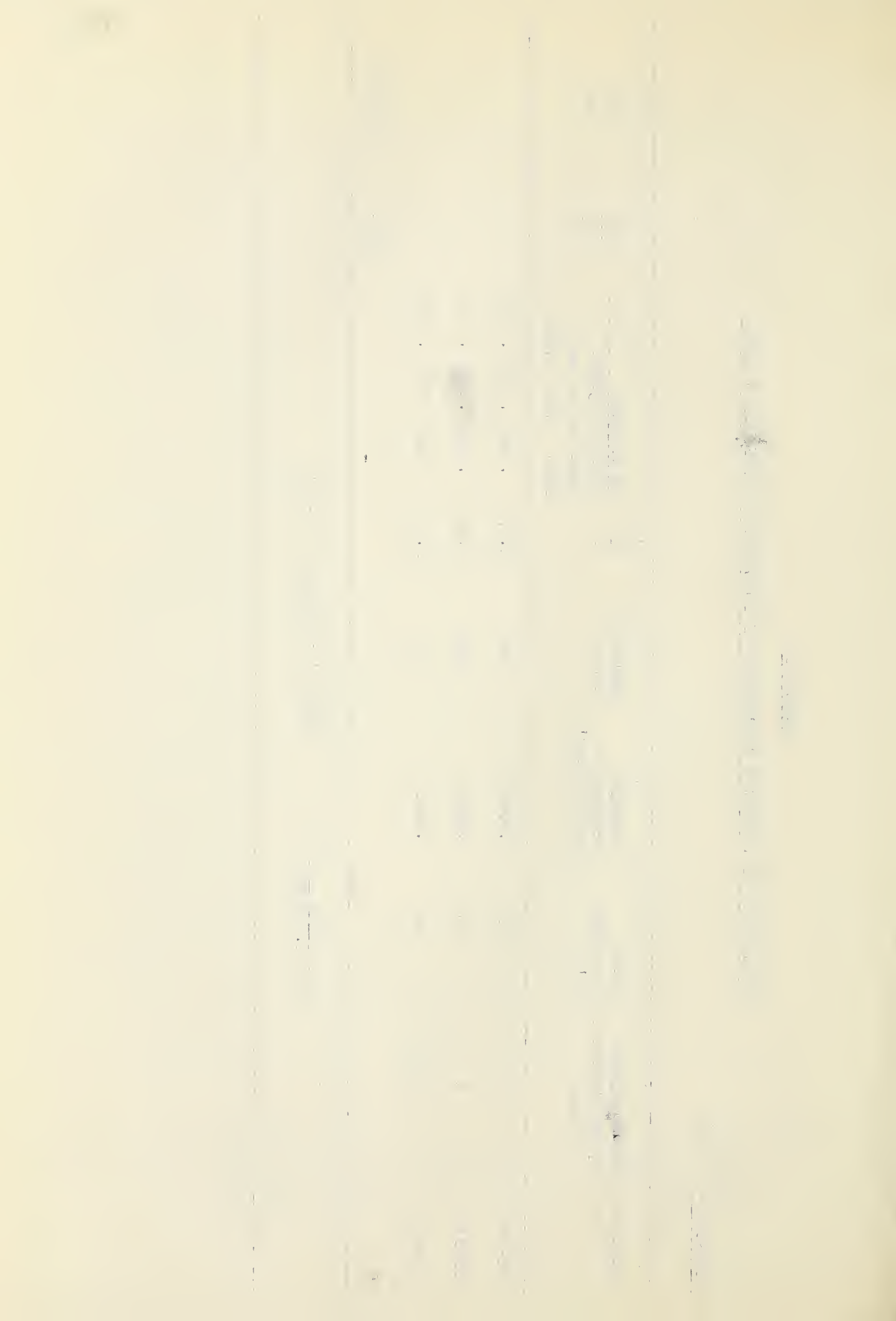
TABLE 15

EVALUATION OF STRENGTH PROPERTIES OF SUBGRADE & REINFORCING LAYER

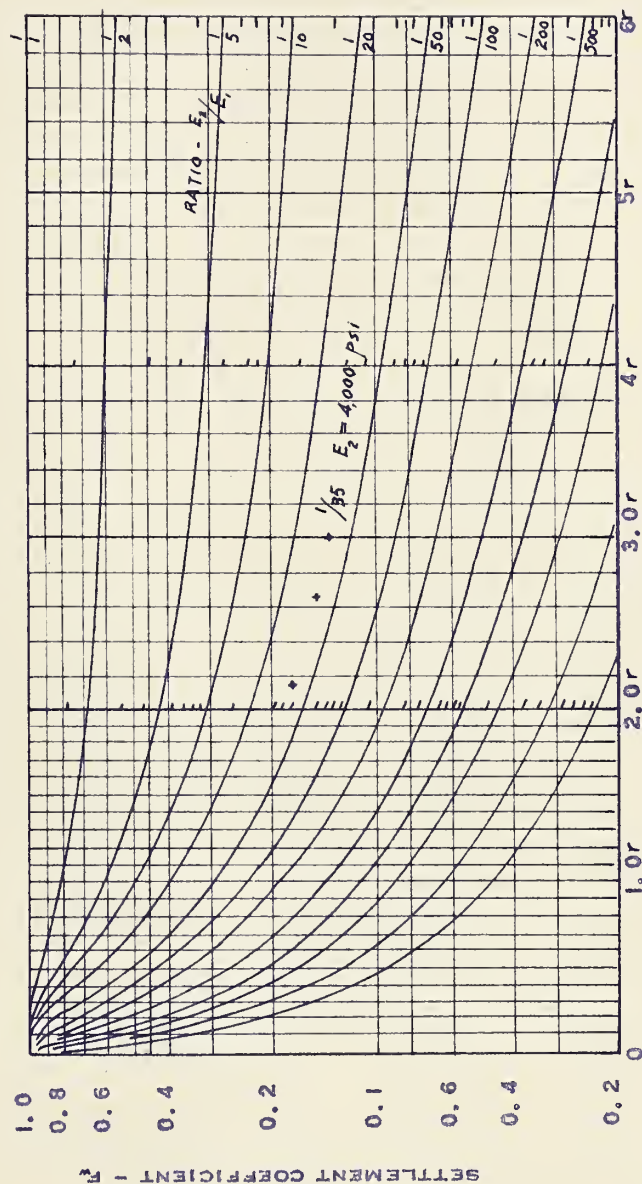
Highway 14 4"A/C

1	2	3	4	5	6	7	8	9	10	11
Section	Chronological Age of Pavement Years	Total Thickness In's	Average Deflection Measured-In's	Number of Trials	$\frac{h}{r}$	Fw values				psi
						Trial Subgrade Modulii, Es psi	3500	4000	4500	
28-29	2	13	.0320	20	2.17	.156	.178	.200		
33-37	1	16	.0276	50	2.66	.134	.153	.173		
31-32	2-1	18	.0265	20	3.00	.129	.147	.166	35	140,000

$$F_w = \frac{W}{1.5 p r} E_s , \quad \text{Where } P = 80 \text{ psi \& } r = 6"$$



EVALUATION OF STRENGTH PROPERTIES OF SUBGRADE AND REINFORCING LAYER



THICKNESS - OF REINFORCING OR PAVEMENT LAYER - 1
EXPRESSED AS MULTIPLES OF THE RADIUS OF BEARING AREA

PLATE 12

EVALUATION OF APPROXIMATE MODULI OF PAVEMENT MATERIALS - I

[DATA FROM TABLE 14]

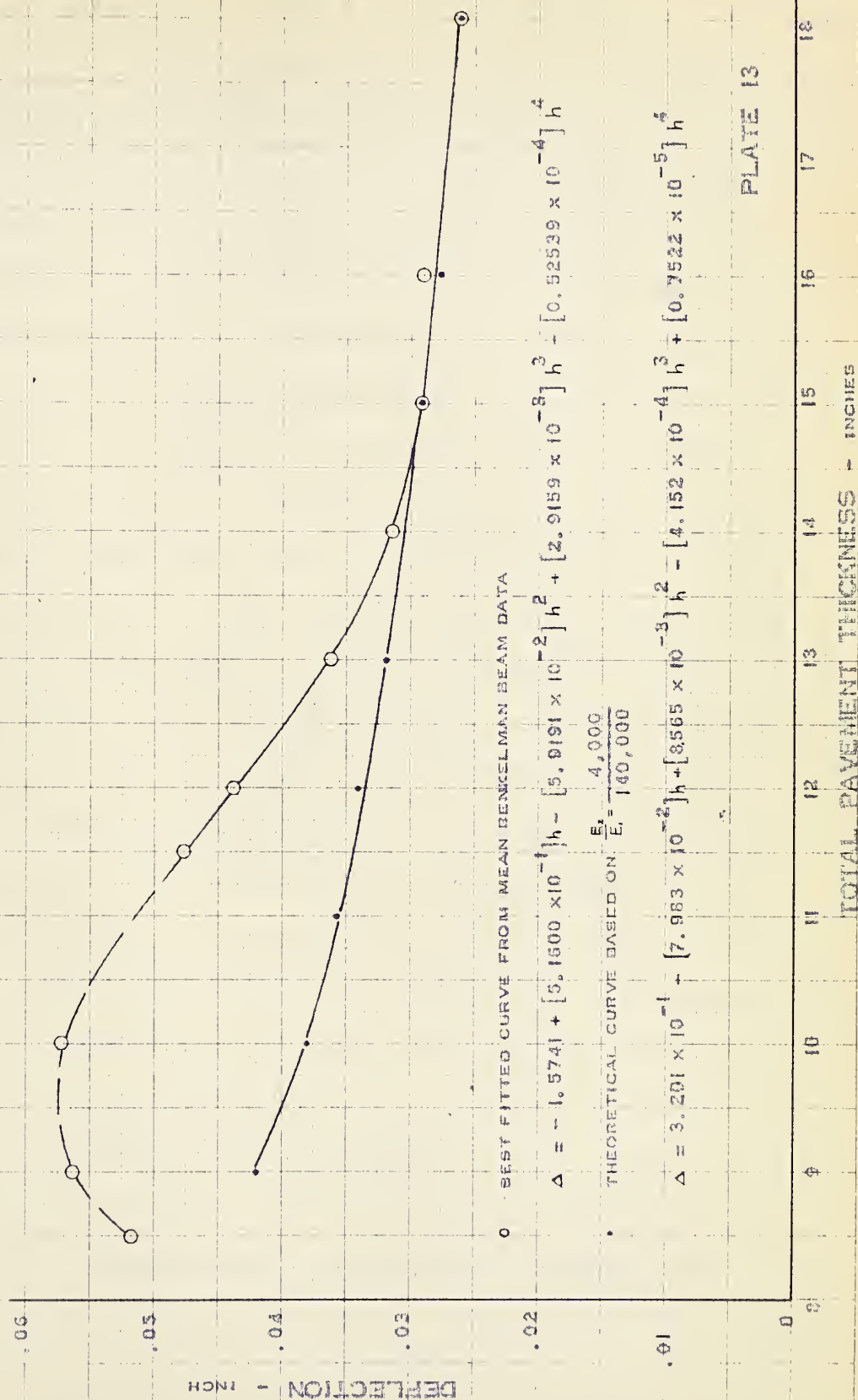


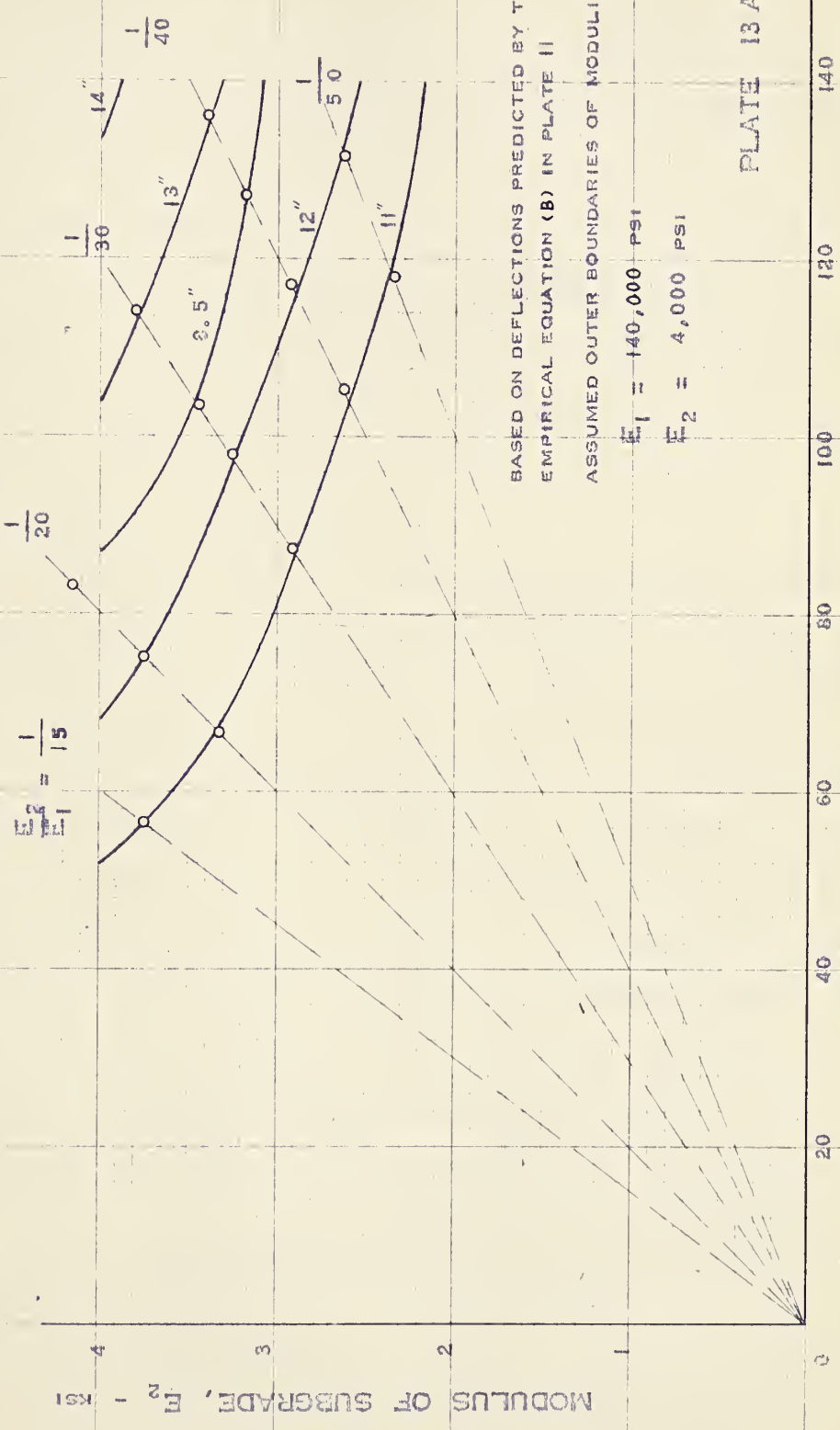
PLATE 13

less than $14\frac{1}{2}$ inches, the two curves are not in agreement in deflection values, and the deviation from each other is shown by the vertical distances at various pavement thicknesses. An attempt was made to correlate this portion of the fitted curve with another theoretical curve based on different moduli lower than those previously obtained. Such an attempt was unsuccessful, because the rate of change in deflection with respect to the change in pavement thickness for the fitted curve was not found to conform with that of the theoretical curves attempted. This disagreement is attributed to the variation in strength properties of the materials in the pavements, resulting from traffic and climatic deterioration over varying periods of service life. Thus, the evaluation of the moduli of the pavement materials is carried out using the deflection values indicated by the best fitted curve (B) at various pavement thicknesses. In the analysis, 4,000 and 140,000 psi are assumed to be the outer boundaries of the modulus values for the subgrade and the reinforcing layer respectively. These moduli are the strength properties of sections having relatively new pavements; therefore, values in excess of these are unlikely for the pavement materials to be investigated.

The curves in Plate 13A show the possible ranges of modulus values for the sections investigated. It is seen that the lowest moduli are approximately 2,180 and 46,000 psi for the subgrade and the reinforcing layer material respectively. It is to be noted that the lowest and the highest moduli values for the combined surfacing

EVALUATION OF APPROXIMATE MODULI OF PAVEMENT MATERIALS - 2

[DATA FROM TABLE 15A]



BASED ON DEFLECTIONS PREDICTED BY THE
EMPIRICAL EQUATION (B) IN PLATE II
ASSUMED OUTER BOUNDARIES OF MODULI

$E_1 = 140,000$ PSI
 $E_2 = 4,000$ PSI

PLATE 13A

and base are within the range obtained by Burmister at WASHO.*

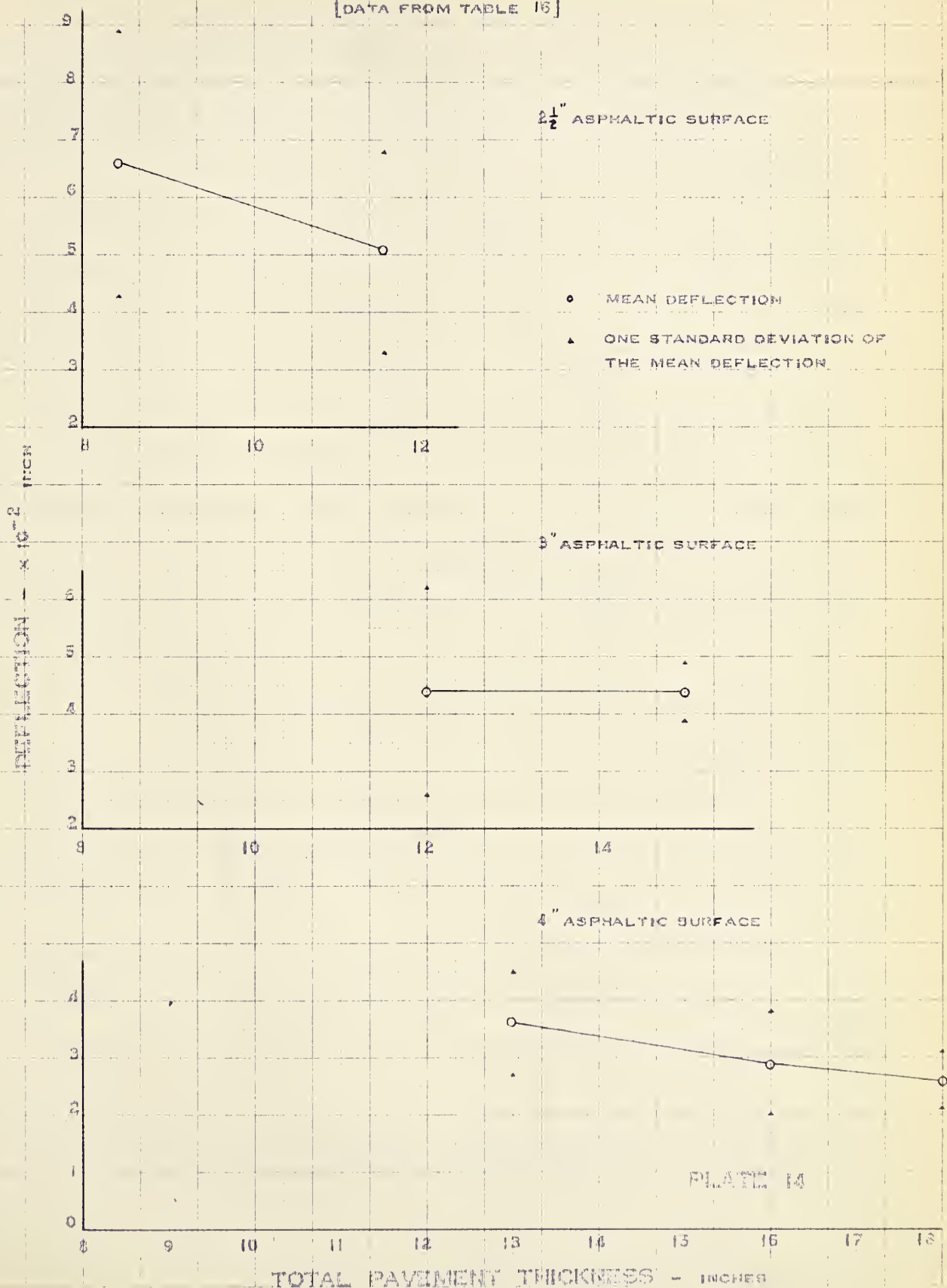
In doing the comparison of the modulus values, it is understood that the WASHO test road is relatively new and that the pavement materials are not necessary strictly comparable. In conjunction with Burmister's deflection equation and the influence curve, the above moduli values so determined appear to be utilizable in the determination of the required thickening of the existing pavement so as to meet the criterion of an arbitrary limiting deflection. Since the possible ranges of the moduli shown by the curves in Plate 13A are based on the predicted deflection from the empirical equation, the accuracy of the estimated moduli is dependent on the goodness of fit of the curve.

Plate 14 groups the sections having the same thickness of bituminous surface. It shows the same trend that the thicker the pavement, the smaller is the deflection. The extent of variability of the deflection values as expressed in standard deviations seems to depend on the magnitude of the deflection. The higher the mean value of deflection of a number of tests, the greater is the variability; and the lower the mean value, the less is the degree of variability. For a section having a total pavement thickness of $8\frac{1}{2}$ inches, the standard deviation is found to be equal to 0.023 inch about the mean deflection of 0.066 inch, while for the section having a total thickness of 18 inches, the standard deviation is 0.005 about the mean value of 0.026 inch. Assuming the deflection data are normally

* See foot note on page 74

DEFLECTION VS PAVEMENT THICKNESS

[DATA FROM TABLE 16]



distributed, 68.27% of the deflection values will be included within the limits of the arithmetic mean deflection values plus and minus one standard deviation($\bar{X} \pm 1.06$)*. Values of mean deflection and standard deviations are tabulated in Table 16.

A large standard deviation about mean deflection indicates a great dispersion of individual measurements about the mean. From an engineering point of view, it may indicate a large number of weak spots in the pavements resulting in inconsistent measurements. Localized failures would be expected in sections having large standard deviations about the high mean deflection values, especially when the pavements are subjected to the detrimental effects of frost heaving and thawing. This suggests that the load ban should be based not only on the information of magnitude of deflection but also on the degree of variation in deflection if localized failure of a pavement is to be prevented.

While the Benkelman beam data obtained from relatively thin pavements show large deflection values, the data may indicate values which are less than the actual deflection in the pavement. In thin pavements some upward deformation of the road surface between the dual tires in deflection tests may occur due to the low slab action produced by the combination of the total thickness of the asphaltic surface and the base course. Consequently the recorded deflection between the tires by the beam would be less than the

* Arkin, H., and Colton, R.R., "Statistical Methods", New York: Barnes & Noble Inc., 1958 ed. (pp. 38).

deflection directly under the tires. This may be one reason accounting for the large variation in deflection values for relatively thin pavements, because the rigidity of these sections to support the loaded vehicle through the slab action of the pavement varies over a larger extent. On the other hand, the measured deflections for relatively thick pavements may show larger values than those occurring directly under the tires, because the lowest point of the deflection bowl occurs between the tires as a result of the slab action of the pavement.

The deflections measured in June, July, and August at various sections are also included in Table 16. There is no significant difference in average deflection from one month to another.

Deflection measurements in accordance with WASHO test procedure of slowly moving wheel load are plotted in Plate 15 to compare the results based on CGRA procedure, which is under static wheel load. It is seen that, based on the limited data, the deflection values yielded by WASHO procedure are lower approximately by 17 per cent than those given by the CGRA procedure, because deflections under static loads are always greater than under moving loads (48).

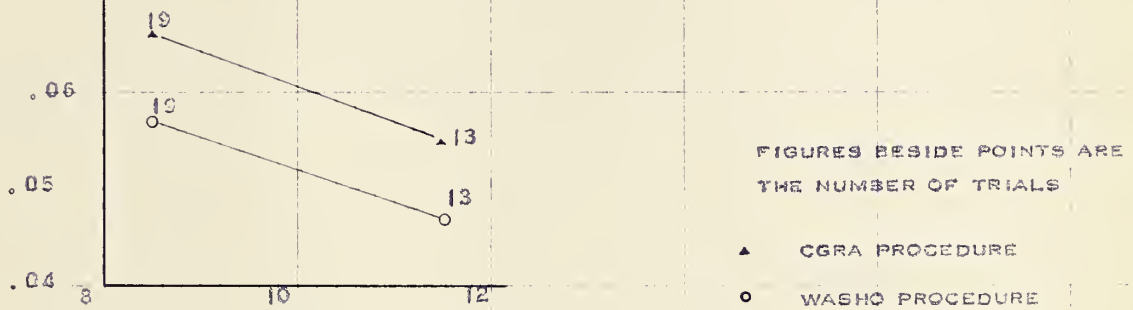
Deflection Versus Chronological Age of The Pavement

Pavements having the same thickness but varying in chronological age are grouped so as to compare the effect of age of pavement on deflection. However, as shown in Plate 16, such an attempt is not too successful. The rapid increase in deflections at

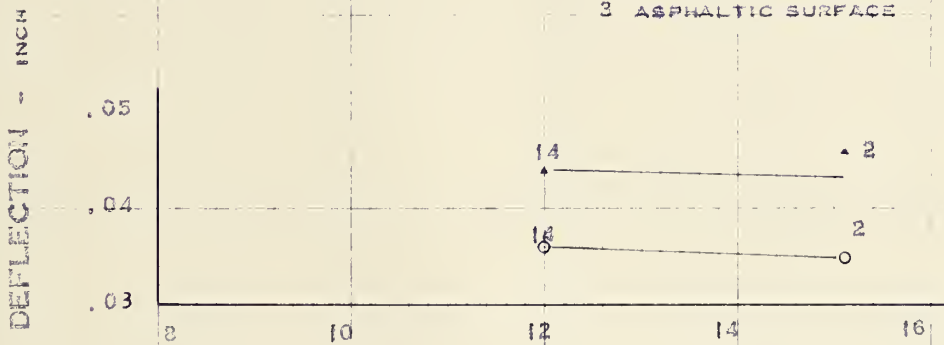
COMPARISON OF DEFLECTIONS MEASURED IN ACCORDANCE WITH CGRA AND WASHO BENKELMAN BEAM PROCEDURES

[DATA FROM TABLE 17]

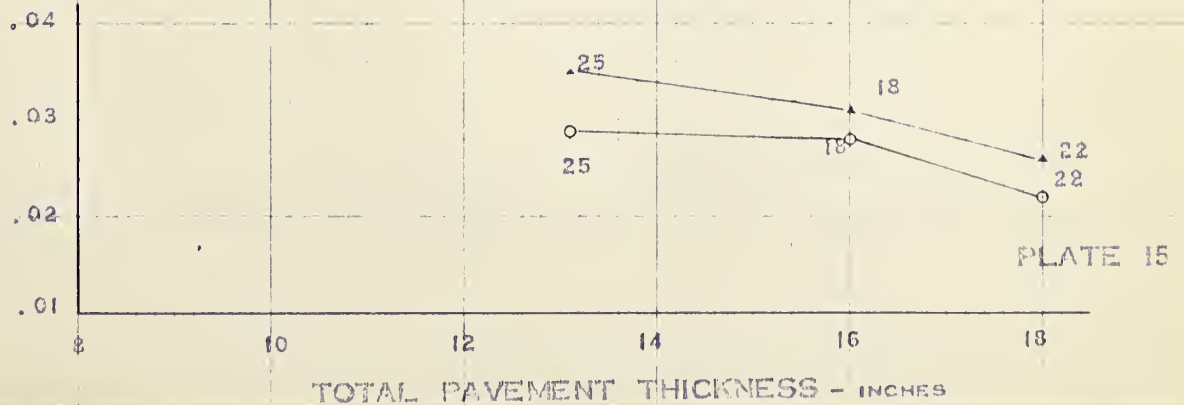
2 $\frac{1}{2}$ " ASPHALTIC SURFACE



3" ASPHALTIC SURFACE



4" ASPHALTIC SURFACE



DEFLECTION vs CHRONOLOGICAL AGE OF PAVEMENT - I

[DATA FROM TABLE 18]

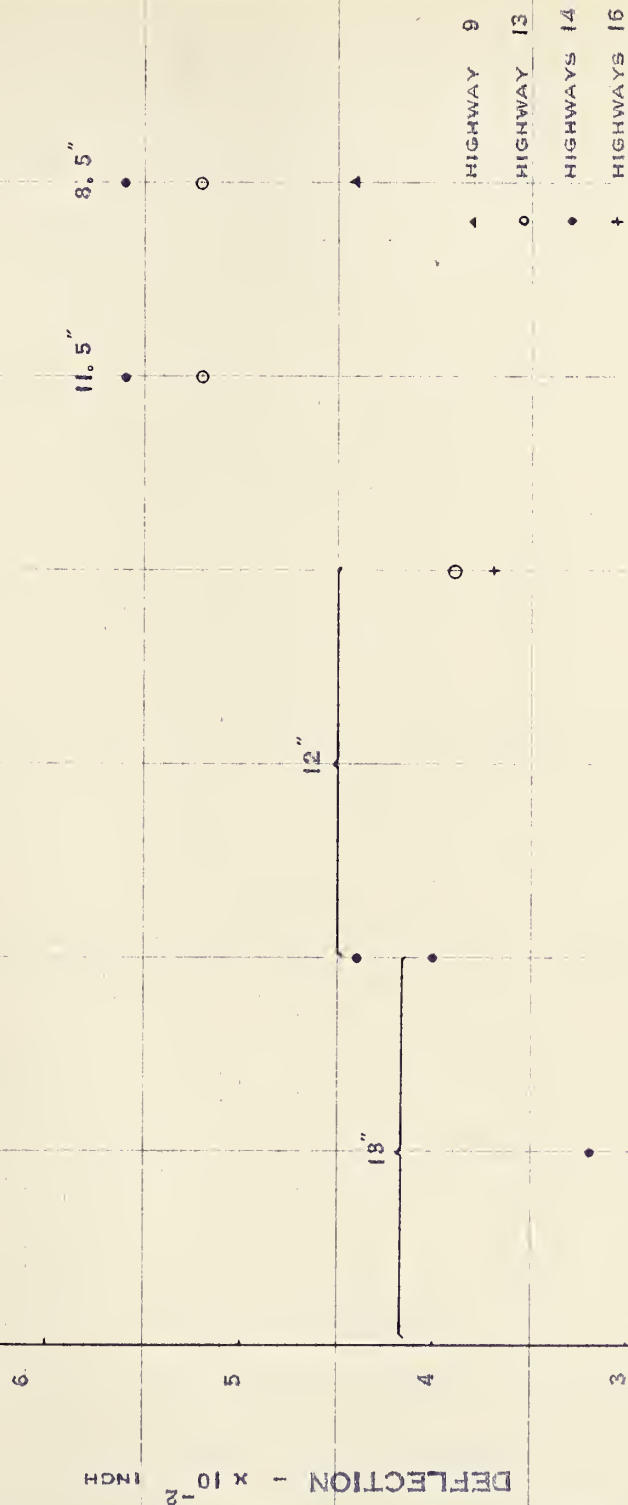


PLATE 16

the sections having pavement thickness of 13 inches may be attributed to the fatigue characteristic of the pavement materials as a result of the effect of moving vehicles during the different durations of service of the pavements considered. This substantiates the relationship between the magnitude of deflection and the number of load repetitions in a pavement: the larger the number of load repetitions, the larger is the total deflection. Because asphalt pavements are undergoing constant change in their physical and chemical properties with the passage of time, the ability to absorb the energy of imposed stresses by the traffic also varies, and thus this change also affects the deflection. In the thinner sections, the spread of points is possibly due to the high variability of the mean deflection values as reflected in high magnitudes of standard deviations as stated. To show the variability of deflections, Plate 18 is made. The curves in the plate show the deflections measured at sections having various pavement thicknesses and different chronological ages. The lines joining the individual points are of no physical significance, and they only serve to show the variation of the values. The figures beside the points denote the pavement temperature when the deflections were taken. The magnitude of the average deflections and the corresponding standard deviations are shown in the plots. Curves B and C in Plate 18 and the histograms in Plate 17 also show the effect of chronological age of pavement on deflection at sections having the same pavement thickness but different in chronological ages. They show that the older the pavement, the larger the deflection,

DEFLECTION vs CHRONOLOGICAL AGE OF PAVEMENT - 2

[DATA FROM TABLE 19]

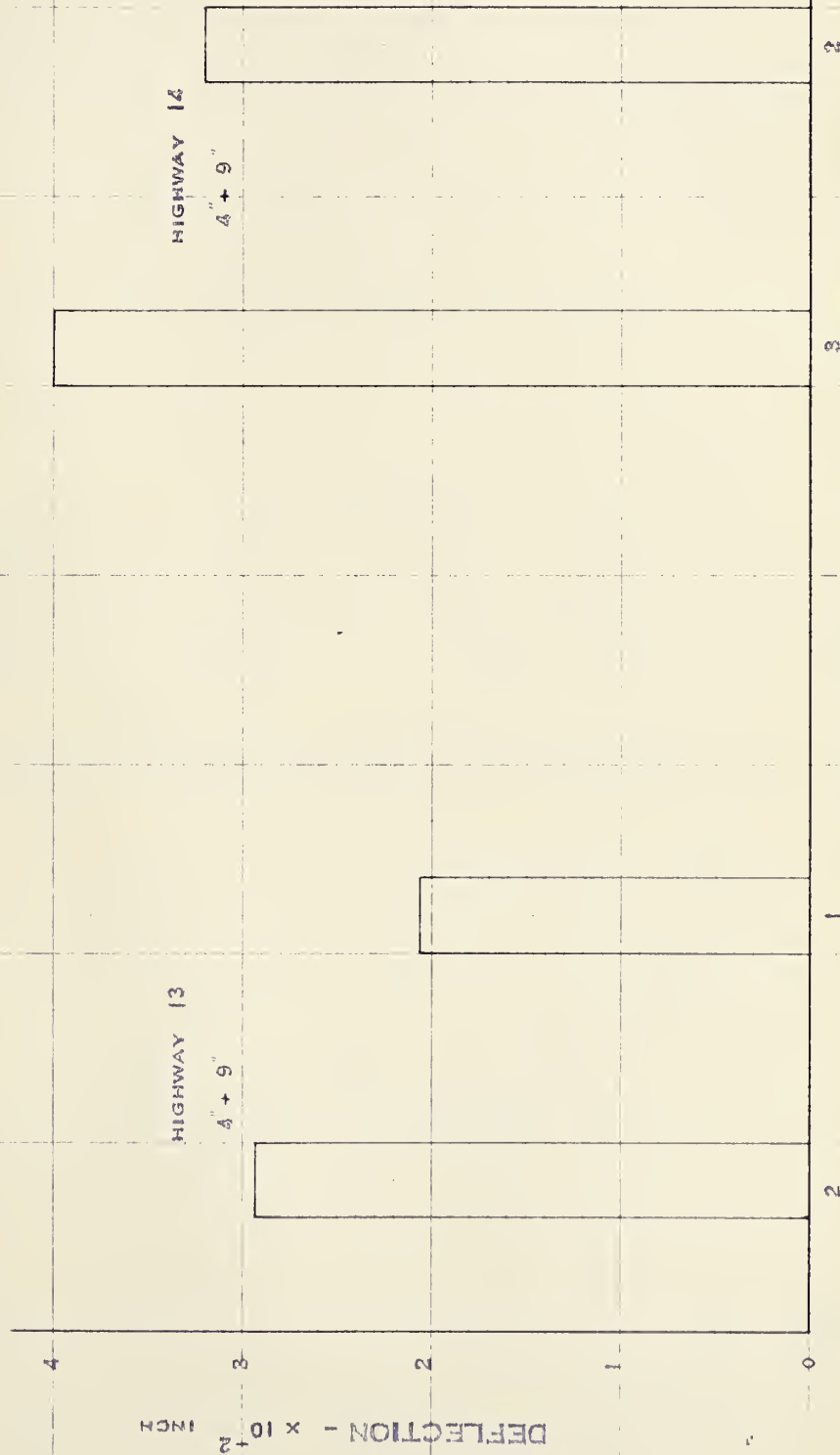
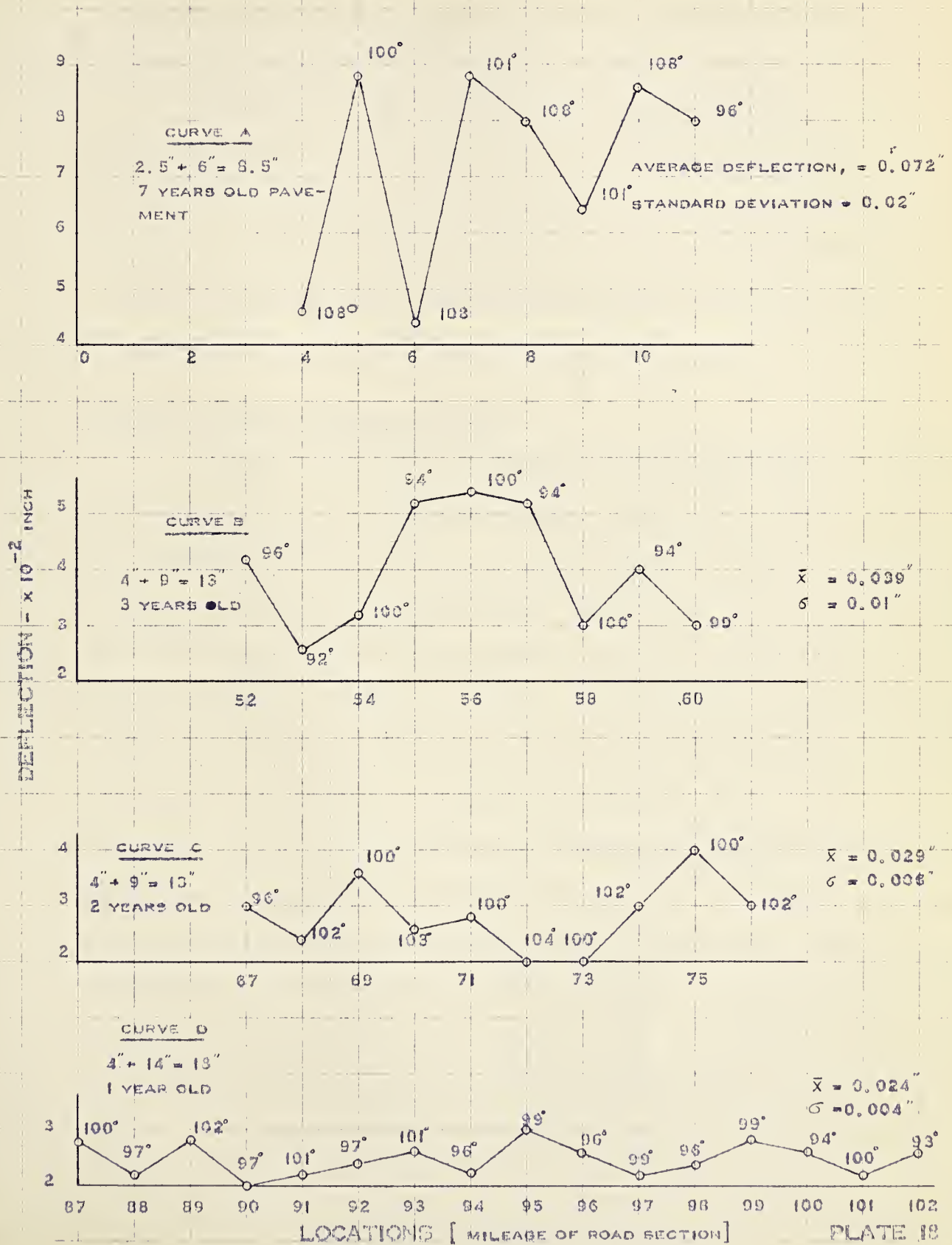


PLATE 17

VARIABILITY OF DEFLECTIONS FOR OLD AND YOUNG PAVEMENTS [DATA FROM TABLE 20]



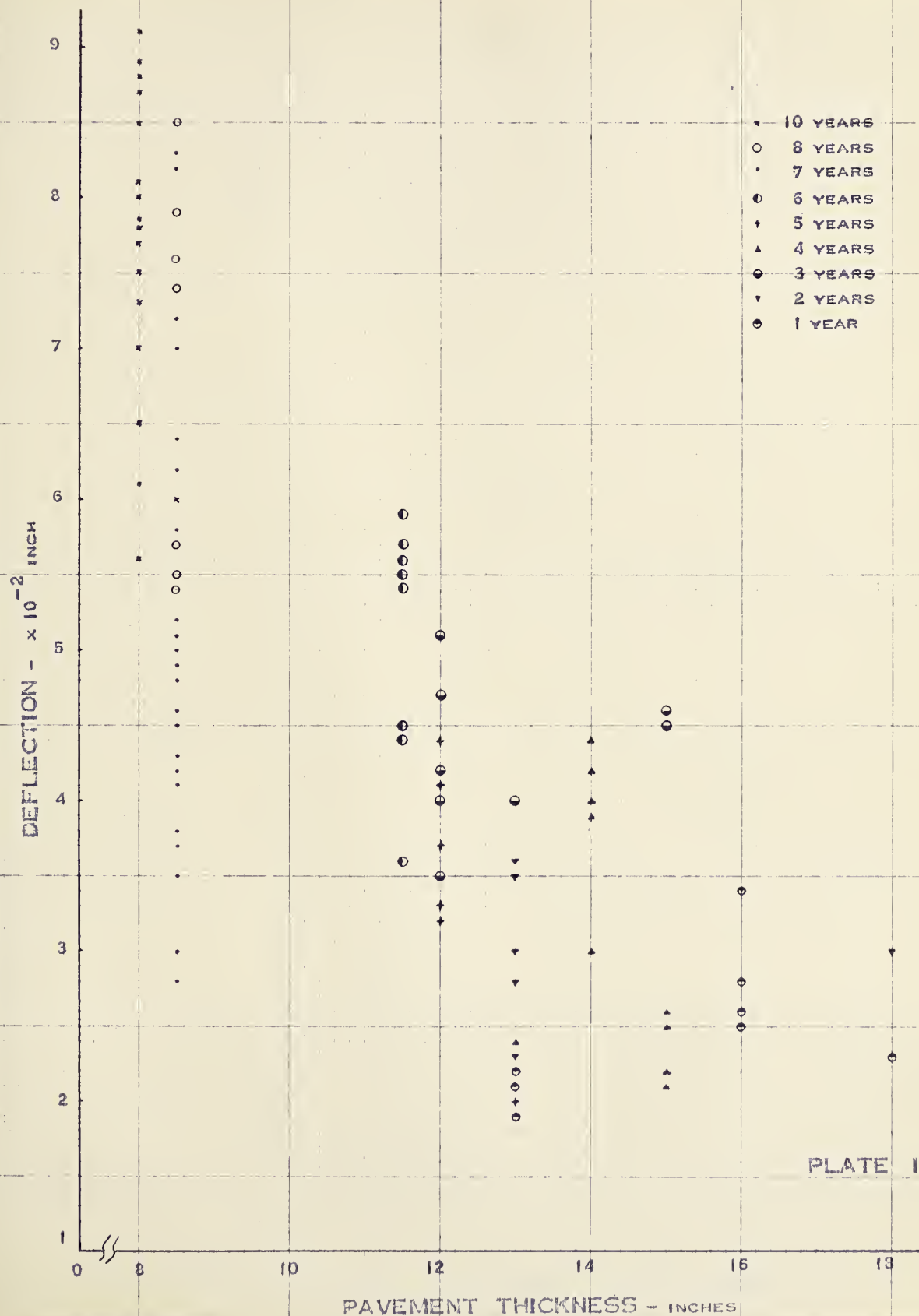
and the larger is the standard deviation about the mean. Curve B shows a standard deviation of 0.01 inch about a mean deflection of 0.039 inch compared with a standard deviation of 0.006 inch about a mean deflection of 0.029 inch, which is shown in Curve C.

Since all the thin pavements are older and exhibit greater deflections than the thick ones, the available data are quite incomplete from a factorial point of view, and consequently no attempt to draw any regression line for the data shown in Plate 19 was made with the aim of uncovering any deflection-thickness-age relationship.

Deflection Versus Subgrade Soil Type

Clays of low to medium plasticity are the predominant subgrade material in the highways studied. Only a few sections have subgrade soil type classified as SP, SC, and CI material according to Casagrande's classification system (29). Based on the limited data, the effect of subgrade soil type on deflection is plotted in Plates 20 and 21. In Plate 20, the heights of the histograms indicate the deflections measured at the sections having the same pavement thickness but different subgrade soil types. It is seen that there is little difference in deflections between sections having a soil type of CL and another of CI; or CH and CI with a total pavement thickness of 13 and 11½ inches respectively. However, the difference in deflections is relatively large in sections having subgrade soil types of CL and SP, and CL and SC. At Highway 9, the deflection measured at the 15-inch section having subgrade soil type of CL is 0.023 inch, while that having a SP subgrade soil is

DEFLECTION-PAVEMENT THICKNESS- PAVEMENT CHRONOLOGICAL AGE [DATA FROM TABLE 21]

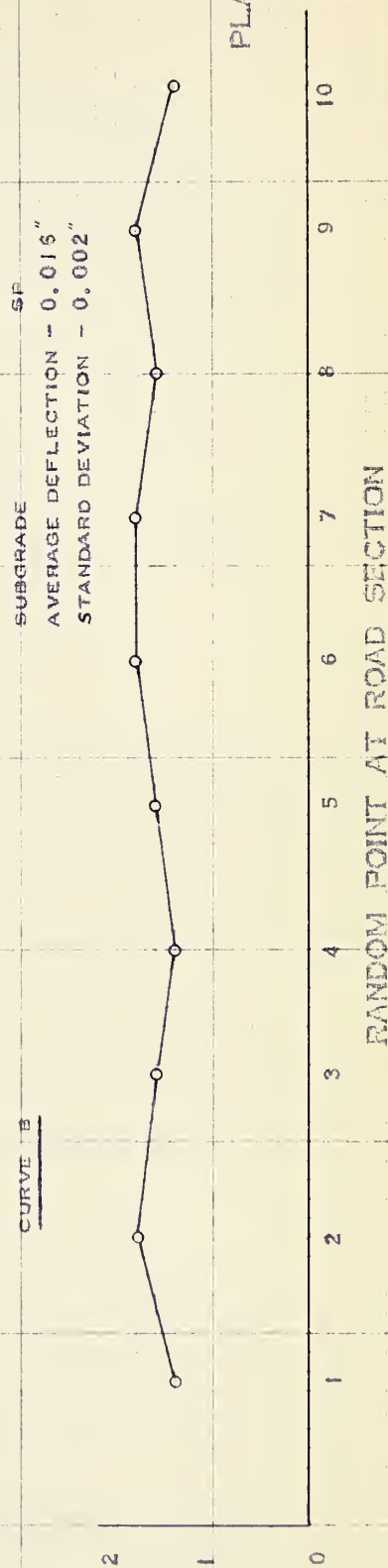
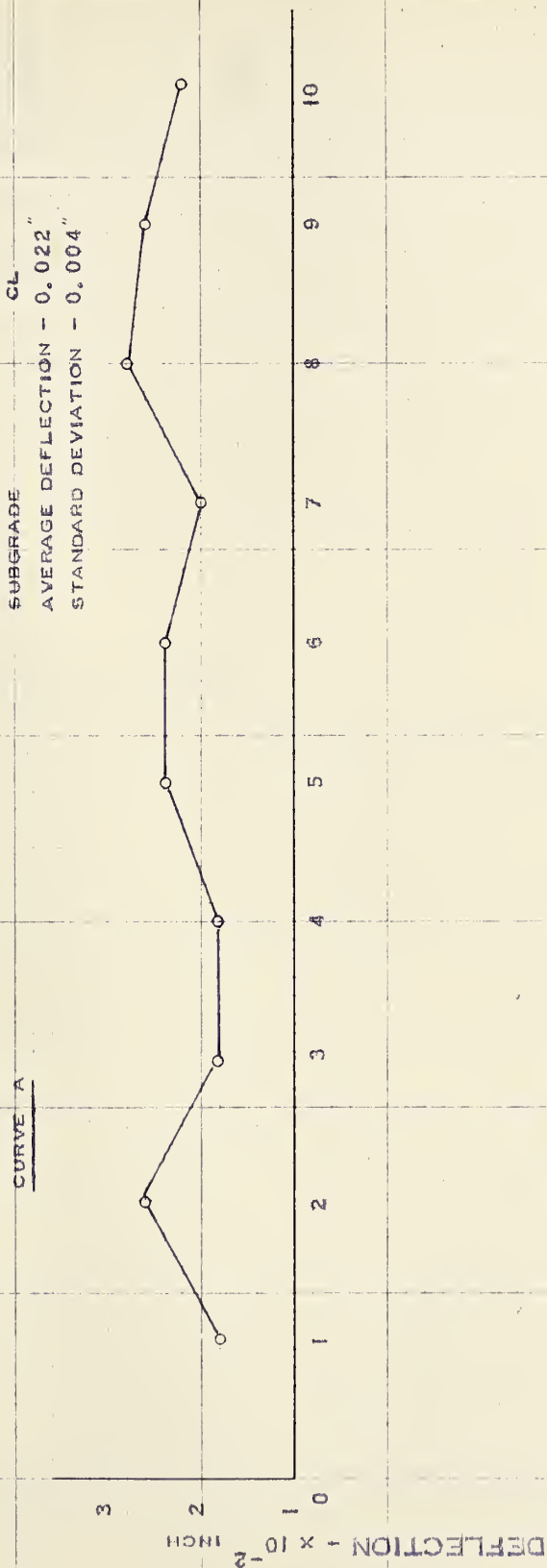


DEFLECTION AND SUBGRADE SOIL TYPE - I

[DATA FROM TABLE 22]



VARIABILITY OF DEFLECTIONS FOR PAVEMENTS WITH DIFFERENT SUBGRADE SOIL TYPES [DATA FROM TABLE 23]



0.016 inch; at Highway 16, the deflections are 0.041 and 0.038 inch, which were measured at the 12-inch sections having CL and SC subgrade soil type.

Plate 21 shows the mean deflection values and their standard deviations for the two sections having the same conditions except different subgrade soil type. The section with a CL subgrade soil exhibits a larger standard deviation about a higher mean deflection than the section with a SP subgrade soil. The mean deflections are 0.022 inch against 0.016 inch and the standard deviations are 0.004 inch versus 0.002 inch for the two sections studied.

Deflection Versus Temperature

Because there is a wide variation in deflection values measured on thinner and older pavements, the investigation of the effect of temperature on deflection is carried out at sections having a young chronological age. Based on this, Plate 22 is made. The deflection measurements were taken at section having a total pavement thickness of 18 inches, with 4 inches of asphaltic surface and 14 inches of base course, and having a chronological age of 1 year. For clarity, only the average deflections at various temperatures are plotted in the plate. Within the temperature range of 58 to 102°F., the relation of deflection and temperature is obtained by the method of least squares. It is expressed by:

$$\Delta = 0.000134 t + 0.0128$$

where ' Δ ' is the deflection and ' t ' is the temperature in degrees

DEFLECTION vs PAVEMENT TEMPERATURE [DATA FROM TABLE 24]

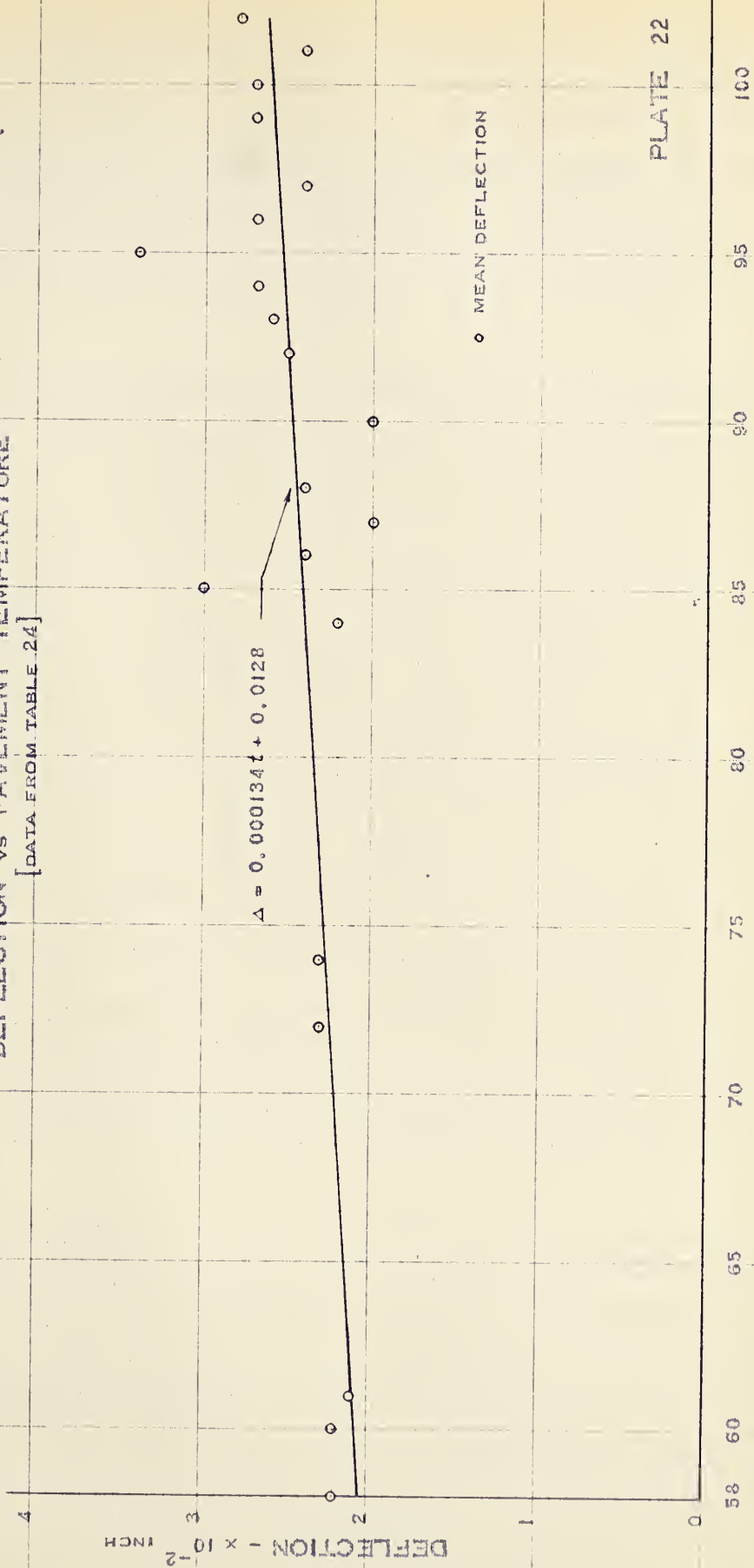


PLATE 22

Fahrenheit. The rate of increase in deflection with increasing temperature as given by this equation is somewhat smaller than that given by the equation obtained by the Canadian Department of Transport* (73), the difference being less than 9 per cent; also, there is a difference in the deflection at 0°F. of 0.00068 inch between the two equations.

The relationship between temperature change and deflection thus obtained is not found to agree with that shown by the curves in Plate 6, the construction of which is based on Nijboer's E-values (41) at various temperatures. It shows, however, a reasonably close agreement with that shown in Plate 9, based on Papazian and Baker's E values (67) at various temperatures. For the 4-inch asphaltic material, the increases in deflection as temperature changes from 41 to 68°F. are found to be equal to 41.4, 15.4 and 19.7 per cent in accordance with the 4-inch curves shown in Plates 6, 9, and the above empirical equation respectively. The 15.4 versus 19.7 per cent for the above-mentioned temperature range and 34 against 44 per cent for a temperature range of 40 to 100°F. from the curve in Plate 9 and the empirical equation may be considered to be reasonable agreement. The above information is tabulated in Table 25 for comparison.

PERFORMANCE RATINGS

The CGRA Special Committee on Pavement Design and Evaluation

* $\Delta = 0.000146 t + 0.01212$

TABLE 25

DEFLECTION INCREASE IN PER CENT FOR A 4-INCH ASPHALTIC MATERIAL
WITH INCREASING TEMPERATURE

SOURCE	INCREASING TEMPERATURE	
	From 41 to 68° F	From 40 - 100° F
Nijboer's E values at various temperatures	41.4	
Papazian and Baker's E values at various temperatures	15.4	34
Empirical Equation, $\Delta = 0.00134 t + 0.0128$	19.7	44

felt (69) that because of implications of variables affecting a pavement structure and of the possibility of facilitating pavement investigation by stages, over-all performance evaluation of existing pavements, constructed according to actual field practice and subjected to various traffic and climatic conditions, is necessary in the correlation with and in the development of a design criterion.

"Present Performance Rating", as adopted by the Committee, is the mean opinion of members of a Rating Panel as to the present ability of a pavement to serve high-speed and high-volume mixed traffic. The Rating Panel is made up of five experienced highway engineers who ride in a fast moving vehicle to assess the riding quality of the pavement in terms of a numerical scale ranging from 0.0 to 10.0 representing extremely poor to extremely good pavements respectively. The rating is primarily concerned with transverse distortion or rutting and with low frequency, high amplitude, and longitudinal distortion of surface, and disregards its geometric design features.

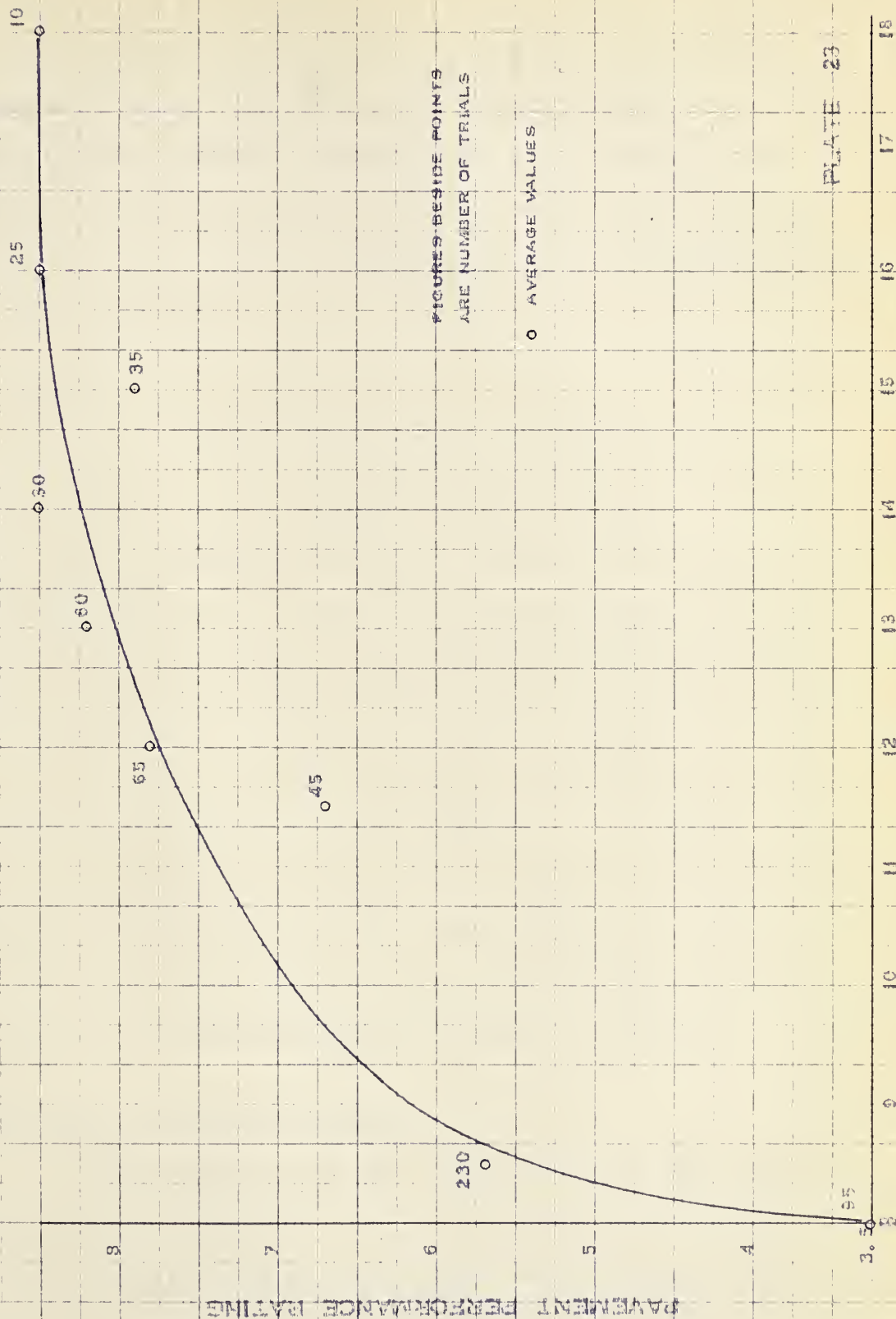
In correlating the performance ratings with other variables, the following items were considered.

Thickness of Total Pavement

Plate 23 shows the relation between rating and pavement thickness. Points in the plot are the average values of many trials at different routes, which are shown by the figures beside individual points. The curve from visual estimation indicates a trend that the rating increases as thicknesses increase, but the rate of increase

PAVEMENT THICKNESS vs PAVEMENT PERFORMANCE RATING

[DATA FROM TABLE 21]



is decreasing as the pavements become thicker. Because the conditions at various sections are not strictly comparable, good correlation cannot be expected. There is a wide variation in ratings at sections having the same thickness at different routes. Generally, thin pavements are older and tend to have lower ratings and wider variations in the rating values as compared with those of thicker pavements, as shown in Table 21.

Deflection

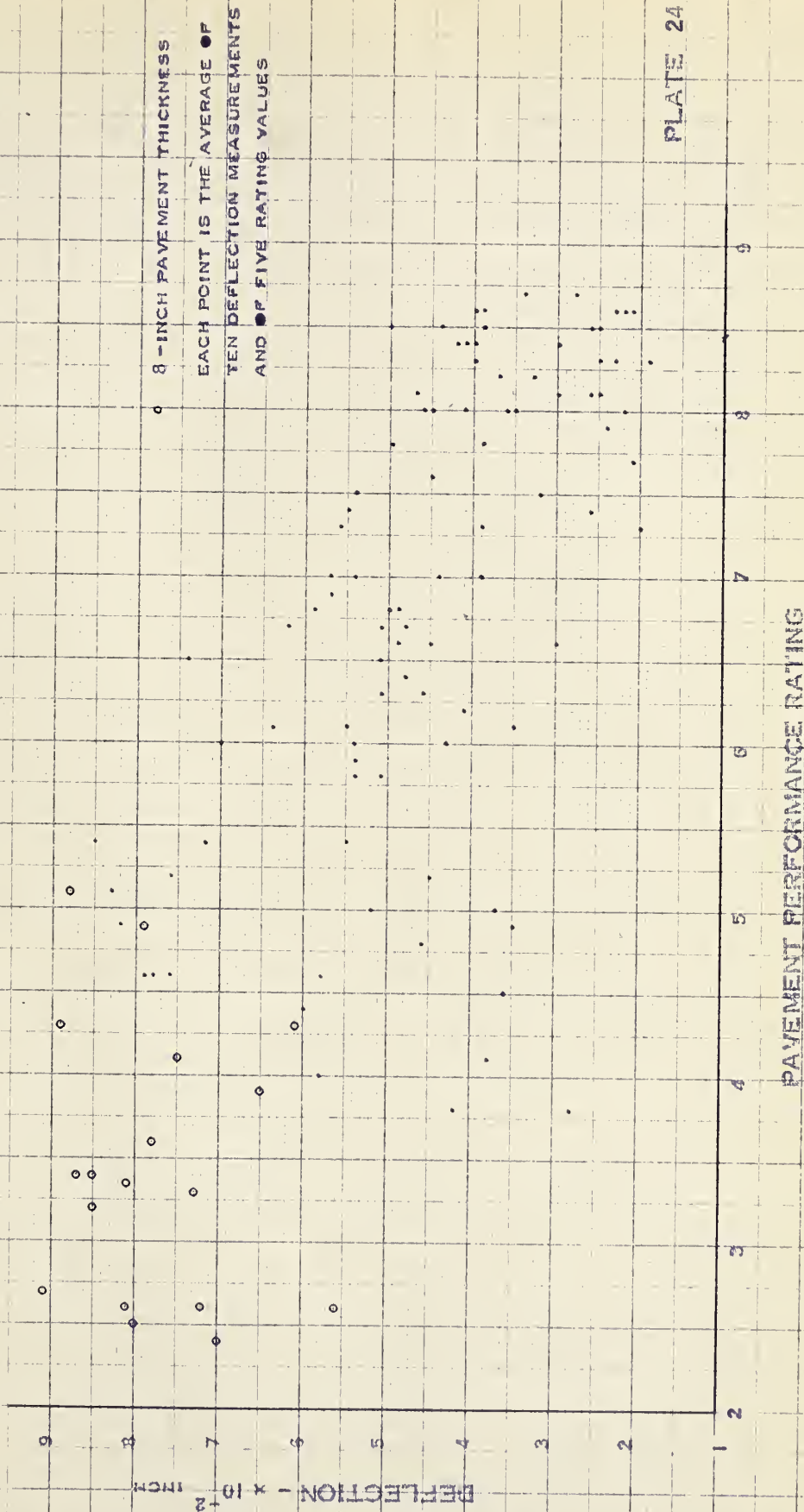
The relation between rating and deflection is shown in Plate 24. Because of a wide spread of points, no regression is drawn. In the examination of the data, variation of values is especially pronounced in sections with thin pavements. For example, the 8-inch section in Highway 16 has a deflection range of 0.091 to 0.056 inch and a rating range from 2.4 to 5.1. The extent of the spread of the deflection and rating values is indicated by the separated plots in Plates 25 to 28 inclusive. It is seen that the sections having thick and young pavements exhibit smaller variations in deflection and rating than those having thin and old pavements. The wide spread of the values is particularly shown by the scatter of points for the 8½-inch pavement at Highway 13 in Plate 26.

Coefficient of Variation of Ratings

Because the sections rated vary from 0.2 to more than 22 miles in length depending on their subdivisions on the basis of homogeneity of conditions, it was thought that the variation in

DEFLECTION vs PAVEMENT PERFORMANCE RATING - I

[DATA FROM TABLE 21]



DEFLECTION VS PAVEMENT PERFORMANCE RATING - 2 [DATA FROM TABLE 26]

HIGHWAYS 9

EACH POINT IS THE AVERAGE
OF TEN DEFLECTION MEASURE-
MENTS AND FIVE RATING VALUES

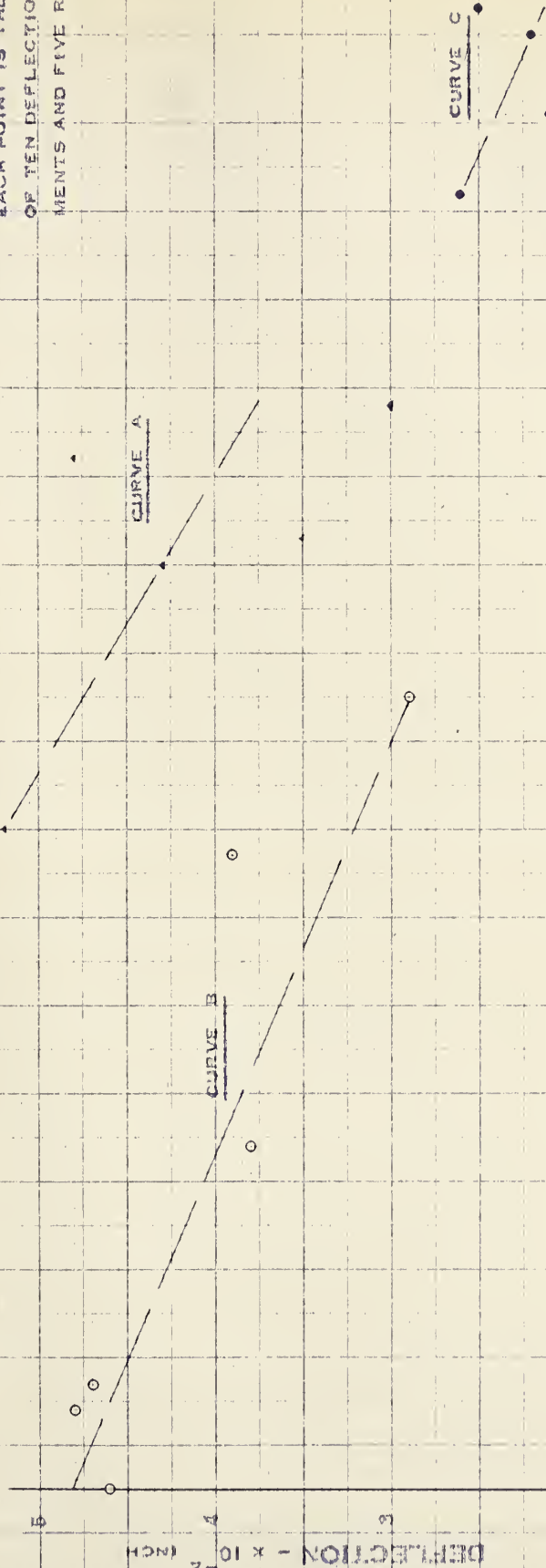
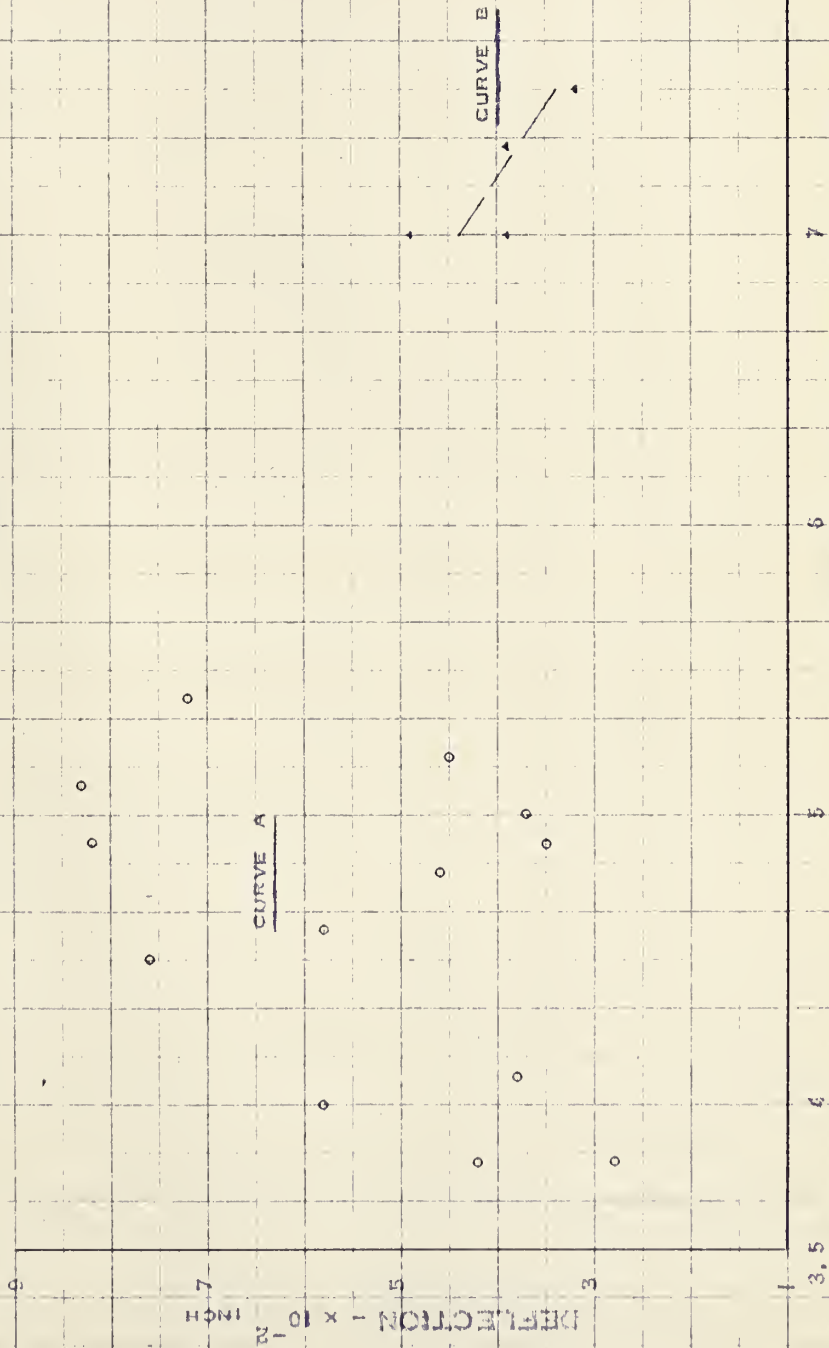


PLATE 25

PAVEMENT PERFORMANCE RATING

DEFLECTION vs PAVEMENT PERFORMANCE RATING - 3

[DATE FROM TABLE 26]



HIGHWAY 13

EACH POINT IS THE AVERAGE OF
TEN DEFLECTION MEASUREMENTS
AND OF FIVE RATING VALUES

PLATE 25

DEFLECTION vs PAVEMENT PERFORMANCE RATING -4

[DATA FROM TABLE 26]

DEFLECTION $\times 10^{-2}$ INCH

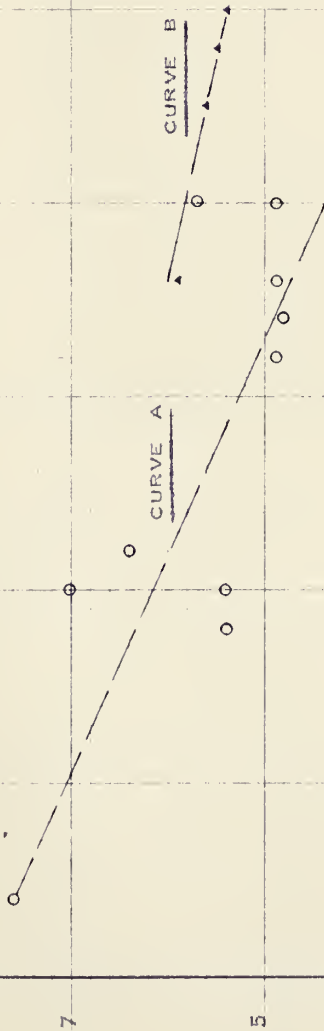
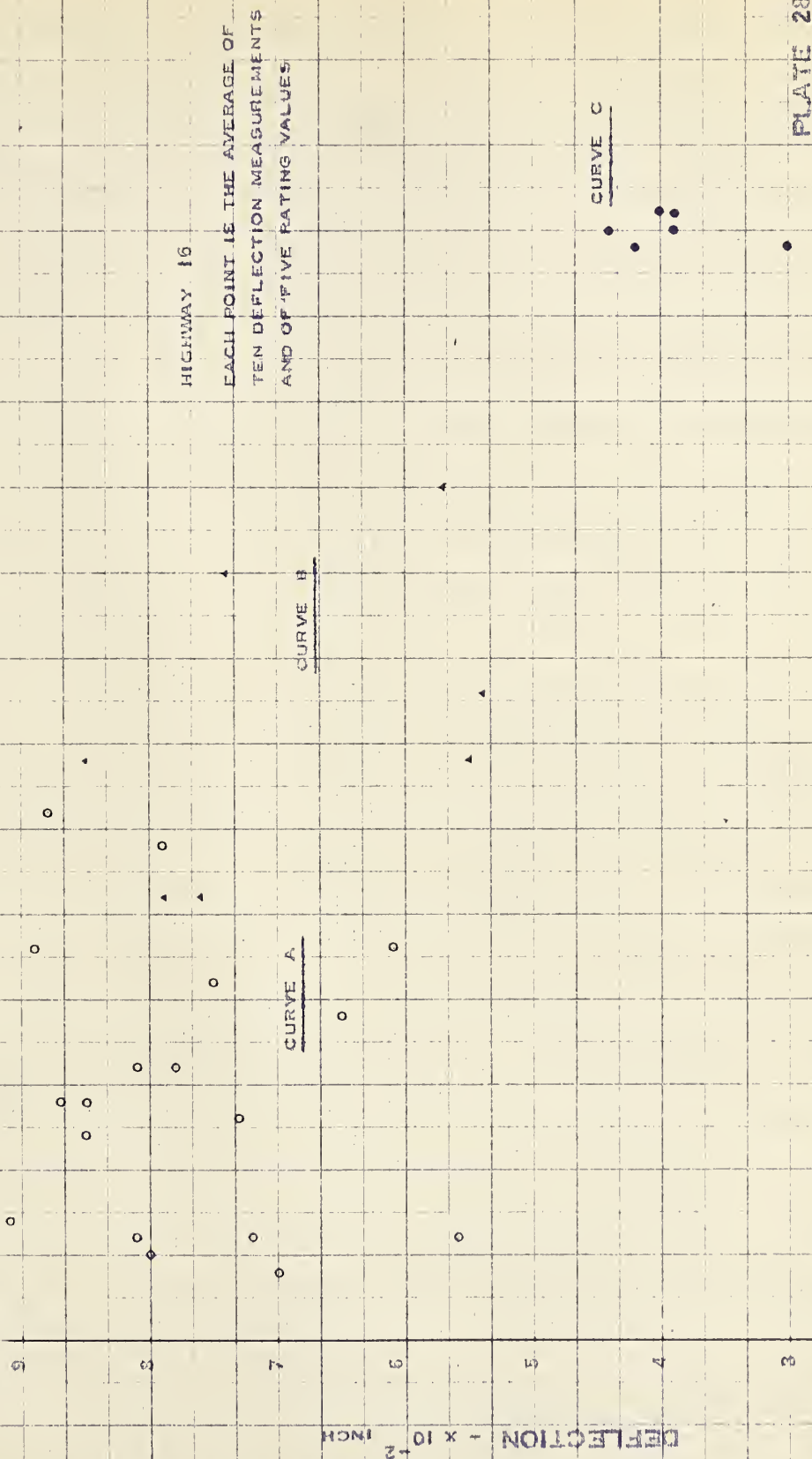


PLATE 27

PAVEMENT PERFORMANCE RATING

DEFLECTION vs. PAVEMENT PERFORMANCE RATING - 5

[DATA FROM TABLE 26]



length might be a factor affecting the inconsistency of the rating values. Plate 29 is intended to show the per cent error or the relative dispersion of the performance ratings with respect to inventory section length. It indicates that the long sections have relatively low and consistent coefficients, and that the short sections, high and inconsistent coefficients. In glancing at the plot, it may seem that the longer is the section, the better is the correlation. However, there is little influence of section length on the mean coefficient since a median line is essentially horizontal. The high and inconsistent values are mainly due to the variations in ratings on the thin sections, which result in wide spread of points in the plot.

The relation between the coefficient and the age of pavement is shown in Plate 30. It is seen that the relative dispersion is smaller with newer pavement as compared with that at the older sections. Because the standard deviations of the rating are low and they vary in a narrow range, and because the mean ratings are high and consistent, the young pavements have low and more consistent coefficients of variation. On the other hand, because a wider variation in standard deviation of the ratings and lower mean ratings, the older pavements have high and inconsistent coefficients. It is believed that because most of the young pavements are thicker than the old ones, the smaller coefficient of variation is also attributable to this factor.

COEFFICIENT OF VARIATION OF PERFORMANCE RATINGS VS SECTION LENGTH OF PAVEMENTS

[DATA FROM TABLE 27]

COEFFICIENT OF VARIATION - PER CENT.

30

25

20

15

10

5

0

σ STANDARD DEVIATION OF RATINGS

\bar{x} AVERAGE VALUE OF RATINGS

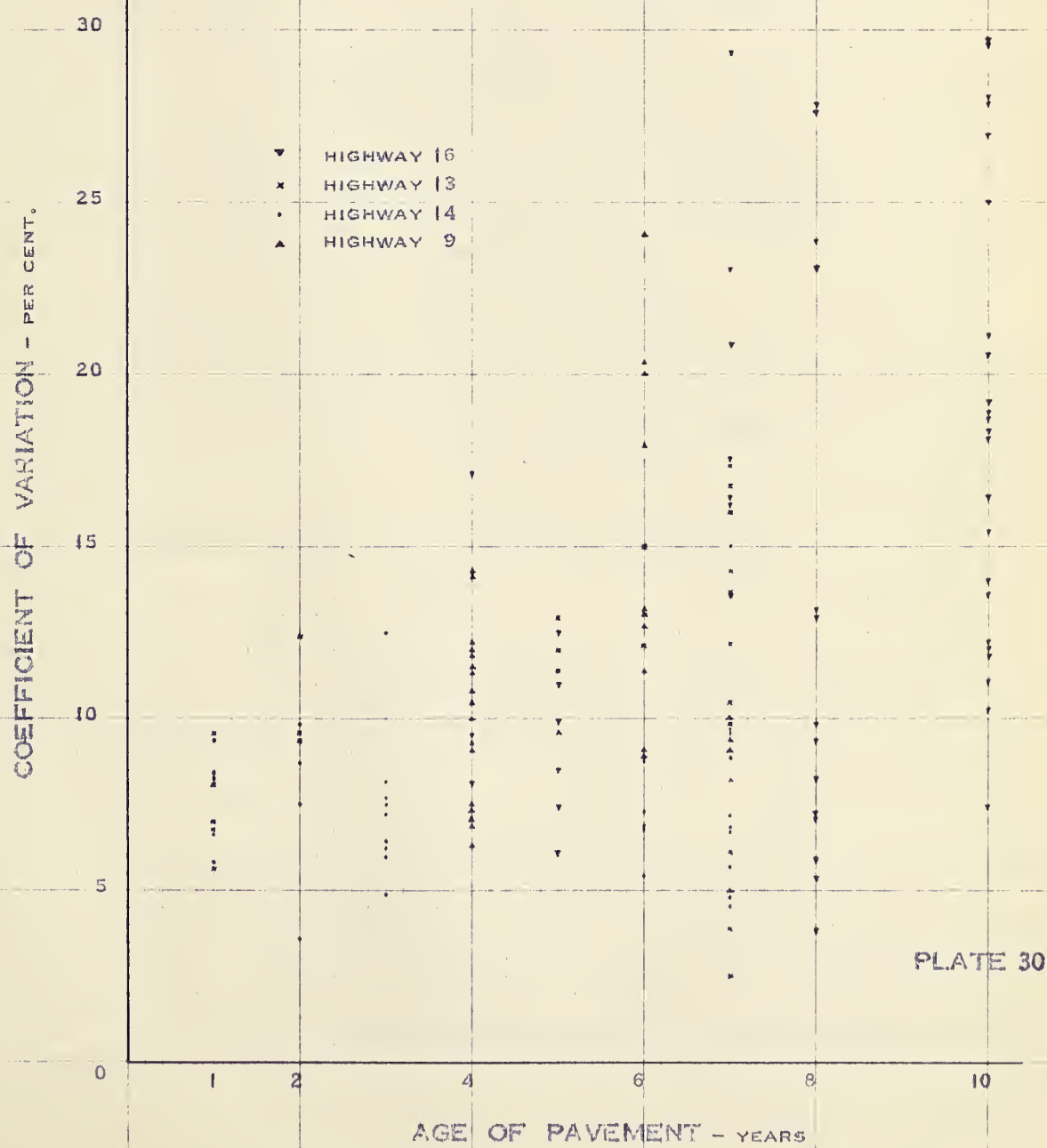
PLATE 29

106

SECTION LENGTH - MILES

10 11 12 13 14 15 22 5

COEFFICIENT OF VARIATION OF PERFORMANCE RATING vs CHRONOLOGICAL AGE OF PAVEMENTS [DATA FROM TABLE 27]



CHAPTER VI

CONCLUSIONS OF THE STUDIES

AND

SUGGESTIONS OF FURTHER INVESTIGATION OF THE TOPIC

CONCLUSIONS

The following conclusions may be drawn from the studies:

1. Strength properties of pavements in terms of moduli can be evaluated from the Benkelman beam data in conjunction with Burmister's deflection equation and the influence curve. For pavements having consistent strength values, the relation of deflection to pavement thickness indicated by the beam data show a close correlation with that determined by Burmister's deflection equation. From the investigation, the moduli of the pavements studied are found to have the following ranges:

- (i) subgrade 2,180 - 4,000 psi.
- (ii) combination of
surfacing and base 46,000 - 140,000 psi.

The moduli are functions of the materials, pavement construction procedures, and the effects of traffic and climate, as reflected in the Benkelman beam data and the pavement performance rating values; the weaker the material, the larger the deflection, and the lower the rating. The above moduli values appear to be utilizable in the determination of the required thickening of the existing pavement so

as to meet the criterion of an arbitrary limiting deflection.

To strengthen a pavement, one may either employ materials having high strength properties or may thicken the pavement to meet an assumed allowable deflection value.

2. For the pavements and subgrade conditions studied and based on the limited available data, the following empirical equations have been evolved for expressing the relationships between the variables:

$$\Delta = \frac{0.47}{h} \quad (1)$$

$$\begin{aligned} \Delta = & -1.5741 + (5.1600 \times 10^{-1})h - (5.9191 \times 10^{-2})h^2 \\ & + (2.9159 \times 10^{-3})h^3 - (0.52539 \times 10^{-4})h^4 \quad (2) \end{aligned}$$

$$\Delta = 0.00134t + 0.0128 \quad (3)$$

Both equations (1) and (2) are the fit of the mean Benkelman beam data obtained on sections having CL subgrade at various temperatures, chronological ages, and performance ratings. Equation (1) is based on the curve fitted by visual estimation and equation (2) is obtained by the use of electronic computer. Because equation (1) has a simple functional form, it appears to have an advantage over the other. Equation (3) is derived from the field measurements on the section having a total pavement thickness of 18 inches, with 4 inches of asphaltic surface and 14 inches of base course, and having a chronological age of 1 year for a temperature range of 58 to 102°F. Due to many variables involved and their unknown relative effects on deflection, the applicability of the above equations is subject to certain limitations and, further verification with more

field data is therefore necessary.

3. In general, relatively thin and old pavements tend to have lower performance ratings, larger deflections, and larger standard deviations about their mean values than relatively thick and new ones. The high standard deviations are significant in the determination of load restrictions to prevent failure or localized failure of a pavement, because they may reflect larger numbers of weak spots in the pavements.

4. Depending on the rigidity of a pavement surface, the Benkelman beam data may record deflections less than the actual pavement deflections directly under the tires as a result of upward deformations of the road surface between the tires.

5. The effect of a tire pressure change of ± 5 psi from a standard tire pressure of 80 psi causes only about 2 per cent variation in surface deflection for the assumed moduli of 4,000 and 120,000 psi for the subgrade and the reinforcing layer material respectively.

6. The available data indicate:

(i) that the deflections of a 13-inch pavement underlain by a subgrade of the CL type is not materially different from a similar pavement underlain by a subgrade of the CI type;

(ii) that the same thing is true of $11\frac{1}{2}$ -inch pavements underlain by subgrades of types CH and CI;

(iii) that the deflections of 15- and 12-inch pavements overlying CL subgrades are greater than those of similar pavements overlying SP or SC subgrades respectively; the former values show a higher standard deviation about a higher mean value than the latter two.

7. Unlike the WASHO deflection test procedure, the CGRA procedure measures the elastic portion of the total deflection of a pavement under a static wheel load. The operation eliminates the possible interference of the probe and the tire walls and minimizes the influence of the deflection pattern on the parts of the deflectometer. Based on the limited data, the deflection values obtained by the WASHO procedure are found to be lower by approximately 17 per cent than those obtained by the CGRA procedure for pavement thickness from $8\frac{1}{2}$ to 18 inches.

SUGGESTIONS

In the development and the application of the beam, it is recommended that further investigation of this topic should include the following points:

1. Correlation of Benkelman beam data with other accepted standard test procedures so as to set up strength criteria of pavement material and to determine the effect of strength on deflection;
2. Evaluation of the variables and their relative effects on the strength of the material and thus on the surface deflection measurements; and
3. Development of a critical surface deflection criterion that conforms with various types of pavements and with desirable performance values.

To achieve these purposes, the necessary work may include:

1. Benkelman beam deflection measurements, including magnitudes of deflections, variabilities of the values, and the shapes of deflection curves for various pavement types and conditions;
2. Correlation of the deflection characteristics with subgrade moisture content, in-place density, field CBR, and plate bearing test results on various component parts of pavements; and
3. Establishment of field and laboratory tests under closely controlled conditions for detailed studies which may include:

- (i) tests on cores and beams cut from asphaltic pavements to investigate the flexibility, fatigue resistance, and modulus of deformation of pavements constructed in accordance with different design criteria and construction procedures, and subjected to different effects of traffic and climatic conditions in various durations of service.
- (ii) tests on prepared specimens and on prototype pavements of different physical properties under various controlled environments, such as thicknesses and types of pavement components, strength of the materials in various layers, sub-surface moisture, temperature, and rate of deformation.

In particular, the work may include the following points:

1. To obtain the deflection bowls of pavements together with the magnitudes of deflection directly under the rear axle of a loaded vehicle in the field;
2. to obtain information on the effect of pavement temperature on deflection in the field at similar pavement structures and on prototype pavements under more closely controlled environments, and

3. to obtain deflection characteristics of a pavement before and after thickening a section of the same; (The thickening can be done by gradually increasing the resurfacing material from a minimum to a certain value. In doing so, the deflections can also be correlated with the actual performance of the section.)
4. to determine the seasonal deflection and performance rating values of various pavements and to observe relative changes in successive years.

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APPENDIX I

TABLE I

EFFECT OF MODULUS OF DEFORMATION OF PAVEMENT MATERIAL AND PAVEMENT THICKNESS ON DEFLECTION
(Plotted in Plate 1)

Thickness h, inches	Curve A		Curve B		Curve C		Curve D		Curve E	
	$\frac{E_2=2,000}{E_1 60,000}$		$\frac{2,000}{100,000}$		$\frac{3,000}{60,000}$		$\frac{4,000}{120,000}$		$\frac{4,000}{60,000}$	
	F _w	Δ	F _w	Δ	F _w	Δ	F _w	Δ	F _w	Δ
9	.266	.0958	.208	.0750	.280	.0673	.252	.0454	.325	.0585
12	.215	.0774	.164	.0590	.232	.0557	.200	.0360	.260	.0468
15	.175	.0630	.138	.0496	.198	.0475	.170	.0306	.233	.0419
18	.150	.0540	.123	.0443	.178	.0427	.150	.0270	.200	.0360
21	.136	.0490	.107	.0385	.160	.0384	.133	.0240	.184	.0331
24	.128	.0461	.095	.0342	.147	.0353	.125	.0225	.168	.0302

1. E1 & E2 - Moduli of Deformation of Reinforcing Material and Subgrade Respectively, psi .

2. F_w - Coefficient of Settlement, Dimensionless.

3. Δ - Surface Deflection, Inch.

TABLE I
ANALYSES OF THE POLYMERIZATION OF VINYL MONOMERS IN THE PRESENCE OF
COPOLYMERIZATION

POLYMERIZATION		COPOLYMERIZATION		ANALYSES	
MONOMER	INITIAL CONCENTRATION	MONOMER	INITIAL CONCENTRATION	ANALYSES	ANALYSES
Styrene	0.100	Styrene	0.100	ANALYSES	ANALYSES
Acrylonitrile	0.100	Acrylonitrile	0.100	ANALYSES	ANALYSES
Methyl Methacrylate	0.100	Methyl Methacrylate	0.100	ANALYSES	ANALYSES
Butadiene	0.100	Butadiene	0.100	ANALYSES	ANALYSES
Isoprene	0.100	Isoprene	0.100	ANALYSES	ANALYSES
1,3-Butadiene	0.100	1,3-Butadiene	0.100	ANALYSES	ANALYSES
2-Methyl-2-Butene	0.100	2-Methyl-2-Butene	0.100	ANALYSES	ANALYSES
2-Pentene	0.100	2-Pentene	0.100	ANALYSES	ANALYSES
3-Pentene	0.100	3-Pentene	0.100	ANALYSES	ANALYSES
2-Hexene	0.100	2-Hexene	0.100	ANALYSES	ANALYSES
3-Hexene	0.100	3-Hexene	0.100	ANALYSES	ANALYSES
2-Heptene	0.100	2-Heptene	0.100	ANALYSES	ANALYSES
3-Heptene	0.100	3-Heptene	0.100	ANALYSES	ANALYSES
2-Octene	0.100	2-Octene	0.100	ANALYSES	ANALYSES
3-Octene	0.100	3-Octene	0.100	ANALYSES	ANALYSES
2-Nonene	0.100	2-Nonene	0.100	ANALYSES	ANALYSES
3-Nonene	0.100	3-Nonene	0.100	ANALYSES	ANALYSES
2-Decene	0.100	2-Decene	0.100	ANALYSES	ANALYSES
3-Decene	0.100	3-Decene	0.100	ANALYSES	ANALYSES
2-Undecene	0.100	2-Undecene	0.100	ANALYSES	ANALYSES
3-Undecene	0.100	3-Undecene	0.100	ANALYSES	ANALYSES
2-Dodecene	0.100	2-Dodecene	0.100	ANALYSES	ANALYSES
3-Dodecene	0.100	3-Dodecene	0.100	ANALYSES	ANALYSES
2-Tridecene	0.100	2-Tridecene	0.100	ANALYSES	ANALYSES
3-Tridecene	0.100	3-Tridecene	0.100	ANALYSES	ANALYSES
2-Tetradecene	0.100	2-Tetradecene	0.100	ANALYSES	ANALYSES
3-Tetradecene	0.100	3-Tetradecene	0.100	ANALYSES	ANALYSES
2-Pentadecene	0.100	2-Pentadecene	0.100	ANALYSES	ANALYSES
3-Pentadecene	0.100	3-Pentadecene	0.100	ANALYSES	ANALYSES
2-Hexadecene	0.100	2-Hexadecene	0.100	ANALYSES	ANALYSES
3-Hexadecene	0.100	3-Hexadecene	0.100	ANALYSES	ANALYSES
2-Heptadecene	0.100	2-Heptadecene	0.100	ANALYSES	ANALYSES
3-Heptadecene	0.100	3-Heptadecene	0.100	ANALYSES	ANALYSES
2-Octadecene	0.100	2-Octadecene	0.100	ANALYSES	ANALYSES
3-Octadecene	0.100	3-Octadecene	0.100	ANALYSES	ANALYSES
2-Nonadecene	0.100	2-Nonadecene	0.100	ANALYSES	ANALYSES
3-Nonadecene	0.100	3-Nonadecene	0.100	ANALYSES	ANALYSES
2-Eicosene	0.100	2-Eicosene	0.100	ANALYSES	ANALYSES
3-Eicosene	0.100	3-Eicosene	0.100	ANALYSES	ANALYSES
2-Heneicosene	0.100	2-Heneicosene	0.100	ANALYSES	ANALYSES
3-Heneicosene	0.100	3-Heneicosene	0.100	ANALYSES	ANALYSES
2-Triacontene	0.100	2-Triacontene	0.100	ANALYSES	ANALYSES
3-Triacontene	0.100	3-Triacontene	0.100	ANALYSES	ANALYSES

ANALYSES OF THE POLYMERIZATION OF VINYL MONOMERS IN THE PRESENCE OF COPOLYMERIZATION

TABLE 1a

APPROXIMATELY CORRESPONDENT VALUES OF MODULUS OF DEFORMATION,
MODULUS OF SUBGRADE REACTION, AND CBR
(For Plate 1)

Modulus of Deformation E2, psi	2,000	2,400	4,000
Modulus of Subgrade Reaction k, pci	113	136	226
California Bearing Ratio, CBR, %	3.4	4.5	14.5

Poisson Ratio equal to 0.5

TABLE 2

EFFECT OF STIFFNESS OF PAVEMENT ON DEFLECTION (Plotted in Plate 2)

DEFLECTION, inch					
$\frac{E_1}{E_2}$	5	10	20	50	100
$\frac{E_2}{F_w}$					
ksi	.38	.27	.20	.14	.10
2.0	.1369	.0972	.0720	.0504	.0360
2.5	.1095	.0778	.0576	.0403	.0288
3.0	.0912	.0648	.0480	.0336	.0240
3.5	.0781	.0555	.0411	.0288	.0206
4.0	.0684	.0486	.0360	.0252	.0182
<div>1. Nomenclature as shown in Table 1.</div> <div>2. Assumed Total Pavement Thickness : 15 inches.</div>					

TABLE 3

EFFECT OF E_2 & E_1 ON PAVEMENT THICKNESS REQUIRED FOR AN ARBITRARY $\frac{E_2}{E_1}$

LIMITING DEFLECTION OF 0.05 INCH

(Plotted in Plate 3)

Total Pavement Thickness h, inches	STIFFNESS RATIO, $\frac{E_2}{E_1}$											
	$\frac{1}{5}$			$\frac{1}{10}$			$\frac{1}{20}$			$\frac{1}{50}$		
	F _w	E2 psi	F _w	E2 psi	F _w	E2 psi	F _w	E2 psi	F _w	E2 psi	F _w	E2 psi
9	.490	7060	.375	5400	.280	4030	.208	3000	.160	2300	.125	1800
11	.440	6340	.325	4680	.240	3460	.175	2520	.134	1930	.108	1555
13	.410	5900	.298	4300	.223	3210	.151	2179	.120	1730	.089	1280
15	.380	5470	.270	3890	.198	2850	.138	1990	.102	1470	.078	1020
17	.355	5110	.248	3570	.180	2590	.125	1800	.090	1295	.069	995
19	.350	4850	.235	3380	.170	2448	.120	1700	.086	1200	.063	900
21	.325	4680	.223	3220	.160	2300	.107	1540	.078	1120	.058	835
24	.310	4470	.210	3020	.145	2090	.095	1370	.070	1010	.051	735

Nomenclature as shown in Table 1.

TABLE 4

$\frac{E_1}{E_2}$ vs PAVEMENT THICKNESS REQUIRED FOR A LIMITING DEFLECTION OF 0.05 INCH AND FOR VARIOUS SUBGRADE MODULI

$\frac{E_1}{E_2}$	$E_2 = 2000 \text{ psi}$		$E_2 = 3000 \text{ psi}$		$\frac{E_1}{E_2}$	$E_2 = 4000 \text{ psi}$	
	$\frac{h}{r}$	$h \text{ inch}$	$\frac{h}{r}$	$h \text{ inch}$		$\frac{h}{r}$	$h \text{ inch}$
70	2.036	12.2	1.50	9.0	20	1.557	9.3
50	2.436	14.6	1.93	11.6	15	1.883	11.3
40	2.818	16.9	2.34	14.0	10	2.370	14.2
30	3.400	20.4	2.80	16.8	8	2.938	17.6
20	4.200	25.2	3.94	23.6	6	3.982	23.9

$$F_w = \frac{E_2 \Delta}{1.5 p r}$$

$p = 80 \text{ psi}$
 $r = 6 \text{ inches}$

TABLE 5

EFFECT OF CHANGE IN MODULUS OF DEFORMATION OF BITUMINOUS MATERIALS
DUE TO TEMPERATURE CHANGES ON SURFACE DEFLECTION -I (Plotted in Plates 5&6)

Temperature ^o F	14	41	68
Modulus of Deformation of Asphalt, E ₁ psi	280,000	70,000	14,000

Nijboer's data (reference 41)

Thickness h, inches	Curve A		Curve B		Curve C	
	t = 68 ^o F		41 ^o F		14 ^o F	
	$\frac{E_2}{E_1} = \frac{4000}{140,000}$		$\frac{4000}{70,000}$		$\frac{4000}{280,000}$	
	F _w	Δ	F _w	Δ	F _w	Δ
2	.94	.169	.82	.148	.62	.117
4	.82	.148	.58	.104	.37	.066
6	.70	.126	.43	.077	.26	.047
9	.60	.108	.31	.056	.18	.032

TABLE 6
EFFECT OF CHANGE IN MODULUS OF DEFORMATION OF BITUMINOUS
MATERIAL DUE TO TEMPERATURE CHANGES ON SURFACE DEFLECTION- II
(Plotted in Plates 7 & 8)

Thickness h, inches	Curve B		Curve C	
	t = .41 OF		t = .14 OF	
	$\frac{E_2}{E_1} = \frac{14000}{70000}$		$\frac{14000}{280000}$	
	F_W	Δ	F_W	Δ
2	.90	.046	.80	.041
3	.82	.042	.66	.034
4	.74	.038	.50	.028
6	.60	.031	.40	.021
7½	.53	.027	.33	.017
9	.49	.125	.29	.014

TABLE 7

EFFECT OF CHANGE IN TEMPERATURE ON SURFACE DEFLECTION
(Plotted in Plate 9)

Temperature OF	40	75	100	140
EI psi	18000	9000	5600	3400

Baker's data (Reference 67)

Thickness h, inches	Curve A		Curve B		Curve C		Curve D	
	t = 40 OF		t = 75 OF		t = 100 OF		t = 140 OF	
	EI = 1700		1700		1700		1700	
	EI 18000		9000		5600		3400	
	F _w	Δ	F _w	Δ	F _w	Δ	F _w	Δ
2	.86	.364	.91	.386	.95	.402	.98	.415
4	.62	.262	.72	.305	.83	.352	.89	.377
6	.47	.199	.60	.250	.72	.305	.81	.344

TABLE 8

EFFECT OF TIRE PRESSURE ON DEFLECTION
(Plotted in Plate 10)

Radius of		h = 9"		h = 11"		h = 15"		h = 19"		h = 24"	
Tire Pressure	contact										
p psi	Area, r inches	$\frac{h}{r}$	F_w	$\frac{h}{r}$	F_w	$\frac{h}{r}$	F_w	$\frac{h}{r}$	F_w	$\frac{h}{r}$	F_w
10	16.90	.53	.540 3.42	.65	.480 3.02	.89	.390 2.47	1.13	.230	2.03	1.42 .270 1.70
20	11.95	.75	.440 3.94	.92	.370 3.32	1.26	.290 2.60	1.59	.240	2.15	2.01 .200 1.79
40	8.45	1.07	.330 4.18	1.30	.280 3.55	1.78	.224 2.80	2.25	.189	2.36	2.84 .155 1.96
60	6.91	1.30	.280 4.36	1.59	.240 3.72	2.17	.190 2.95	2.75	.160	2.48	3.48 .135 2.10
80	5.98	1.50	.252 4.55	1.84	.220 3.94	2.51	.170 3.06	3.18	.145	2.63	4.01 .124 2.23
100	5.35	1.68	.236 4.75	2.06	.200 4.02	2.81	.160 3.21	3.56	.136	2.75	4.48 .117 2.36

$$\Delta = \frac{1.5 p r F_w}{E_2} ; \quad \text{in x } 10^{-2} \text{ inch}$$

Assumed values: $E_2 = 4000$ psi & $E_1 = 120,000$ psi

TABLE 13

DEFLECTION - PAVEMENT THICKNESS (Plotted in Plate 11)

Route	Pavement thickness inches	Average rating	Average Deflection inch	Number of Trials	Deflection based on fitted curve (A), inch	Deflection based on fitted curve (B), inch
9, 14, 16	8½	6.5	.0516	230	.0553	.0518
9, 13, 14	11½	7.0	.0530	80	.0409	.0477
13, 14, 16	12	7.8	.0405	130	.0392	.0436
9, 13, 14	13	8.2	.0285	110	.0362	.0362
16	14	8.5	.0390	60	.0336	.0314
9, 14	15	7.9	.0296	70	.0314	.0293
14	16	8.5	.0273	60	.0294	.0290
14	18	8.5	.0265	20	.0261	.0262
	9				.0523	.0564
	10				.0470	.0573

Fitted curve (A) $\Delta = \frac{0.47}{h}$

Fitted curve (B) $\Delta = -1.5741 + (5.1600 \times 10^{-1})h - (5.9191 \times 10^{-2})h^2 + (2.9159 \times 10^{-3})h^3 - (0.52539 \times 10^{-4})h^4$



TABLE 14

EVALUATION OF APPROXIMATE MODULII OF PAVEMENT
MATERIALS (Plotted in Plate 13)

Pavement Thickness h, inches	Deflection from the fitted curve (B) inch	$\frac{E_2}{E_1} = \frac{4000}{140000} = 1/35$ F_w	Δ
8½	.0518		
9	.0564	.234	.0421
10	.0573	.0211	.0380
11	.	.0200	.0360
11½	.0477		
12	.0436	.190	.0342
13	.0362	.178	.0320
14	.0314		
15	.0293	.164	.0295
16	.0290	.153	.0276
18	.0262	.147	.0264

TABLE 15A
EVALUATION OF APPROXIMATE MODULI OF PAVEMENT
MATERIALS (Plotted in plate 13A)

Pavement Thickness h, inches	Predicted Deflection inch	$\frac{E_2}{E_1}$									
		1/15	1/20	1/30	1/40	1/50					
		F_w psi	E_2 psi	F_w psi	E_2 psi	F_w psi	E_2 psi	F_w psi	E_2 psi	F_w psi	E_2 psi
8½	.052		130	4150	.25	3460	.23	3180	.22	3050	
11	.052	.27	3740	.24	3320	.21	2910	.19	2630	.17	2360
12	.044			.23	3760	.20	3270	.18	2940	.16	2620
13	.036			.22	4400	.19	3800	.17	3400	.15	3000
14	.031					.18	4180	.16	3720		

TABLE I				SUMMARY OF RESULTS	
Year	Month	Day	Time	Location	Remarks
1911	Jan	1	10:00	St. Paul	First day of work
1911	Jan	2	10:00	St. Paul	Second day of work
1911	Jan	3	10:00	St. Paul	Third day of work
1911	Jan	4	10:00	St. Paul	Fourth day of work
1911	Jan	5	10:00	St. Paul	Fifth day of work
1911	Jan	6	10:00	St. Paul	Sixth day of work
1911	Jan	7	10:00	St. Paul	Seventh day of work
1911	Jan	8	10:00	St. Paul	Eighth day of work
1911	Jan	9	10:00	St. Paul	Ninth day of work
1911	Jan	10	10:00	St. Paul	Tenth day of work
1911	Jan	11	10:00	St. Paul	Eleventh day of work
1911	Jan	12	10:00	St. Paul	Twelfth day of work
1911	Jan	13	10:00	St. Paul	Thirteenth day of work
1911	Jan	14	10:00	St. Paul	Fourteenth day of work
1911	Jan	15	10:00	St. Paul	Fifteenth day of work
1911	Jan	16	10:00	St. Paul	Sixteenth day of work
1911	Jan	17	10:00	St. Paul	Seventeenth day of work
1911	Jan	18	10:00	St. Paul	Eighteenth day of work
1911	Jan	19	10:00	St. Paul	Nineteenth day of work
1911	Jan	20	10:00	St. Paul	Twentieth day of work
1911	Jan	21	10:00	St. Paul	Twenty-first day of work
1911	Jan	22	10:00	St. Paul	Twenty-second day of work
1911	Jan	23	10:00	St. Paul	Twenty-third day of work
1911	Jan	24	10:00	St. Paul	Twenty-fourth day of work
1911	Jan	25	10:00	St. Paul	Twenty-fifth day of work
1911	Jan	26	10:00	St. Paul	Twenty-sixth day of work
1911	Jan	27	10:00	St. Paul	Twenty-seventh day of work
1911	Jan	28	10:00	St. Paul	Twenty-eighth day of work
1911	Jan	29	10:00	St. Paul	Twenty-ninth day of work
1911	Jan	30	10:00	St. Paul	Thirtieth day of work
1911	Jan	31	10:00	St. Paul	Thirty-first day of work
1911	Feb	1	10:00	St. Paul	First day of work
1911	Feb	2	10:00	St. Paul	Second day of work
1911	Feb	3	10:00	St. Paul	Third day of work
1911	Feb	4	10:00	St. Paul	Fourth day of work
1911	Feb	5	10:00	St. Paul	Fifth day of work
1911	Feb	6	10:00	St. Paul	Sixth day of work
1911	Feb	7	10:00	St. Paul	Seventh day of work
1911	Feb	8	10:00	St. Paul	Eighth day of work
1911	Feb	9	10:00	St. Paul	Ninth day of work
1911	Feb	10	10:00	St. Paul	Tenth day of work
1911	Feb	11	10:00	St. Paul	Eleventh day of work
1911	Feb	12	10:00	St. Paul	Twelfth day of work
1911	Feb	13	10:00	St. Paul	Thirteenth day of work
1911	Feb	14	10:00	St. Paul	Fourteenth day of work
1911	Feb	15	10:00	St. Paul	Fifteenth day of work
1911	Feb	16	10:00	St. Paul	Sixteenth day of work
1911	Feb	17	10:00	St. Paul	Seventeenth day of work
1911	Feb	18	10:00	St. Paul	Eighteenth day of work
1911	Feb	19	10:00	St. Paul	Nineteenth day of work
1911	Feb	20	10:00	St. Paul	Twentieth day of work
1911	Feb	21	10:00	St. Paul	Twenty-first day of work
1911	Feb	22	10:00	St. Paul	Twenty-second day of work
1911	Feb	23	10:00	St. Paul	Twenty-third day of work
1911	Feb	24	10:00	St. Paul	Twenty-fourth day of work
1911	Feb	25	10:00	St. Paul	Twenty-fifth day of work
1911	Feb	26	10:00	St. Paul	Twenty-sixth day of work
1911	Feb	27	10:00	St. Paul	Twenty-seventh day of work
1911	Feb	28	10:00	St. Paul	Twenty-eighth day of work
1911	Feb	29	10:00	St. Paul	Twenty-ninth day of work
1911	Feb	30	10:00	St. Paul	Thirtieth day of work
1911	Feb	31	10:00	St. Paul	Thirty-first day of work

1911

TABLE 16

DEFLECTION MEASURED IN ACCORDANCE WITH CGRA PROCEDURE
(Plotted in Plate 14)

Highway 14															
Section	Pavement Thickness Age of inches		June			July			August			Average			
	Surface	Base Total Pavement Years	1	2	3	N	t	Δ	N	t	Δ	N	t	Δ	σ ⁴
			N	t	Δ	N	t	Δ	N	t	Δ	N	t	Δ	σ ⁴
3-13	2½	6	8½	7	19	81	6.5	19	102	6.6	19	93	6.6	2.3	
14-17	2½	9	11½	6	13	82	45	13	95	5.5	13	81	5.5	1.8	
18-22	3	9	12	3	14	83	4.1	14	93	4.4	14	69	4.5	1.8	
24-25	3	12½	13	3	2	86	4.1	2	93	4.1	2	75	4.7	0.5	
26-29	4	9	13	3-2	25	86	3.5	25	99	3.5	25	80	3.8	0.9	
30,33-37	4	12	16	2-1	19	88	2.8	18	98	3.1	19	96	2.9	0.9	
31-32	4	14	18	2-1	32	87	2.4	32	98	2.6	22	95	2.7	0.5	

1. N denotes number of trials

2 t denotes pavement temperature in °F

3 Δ denotes surface deflection in $\times 10^{-2}$ inch

4 standard deviations of deflection in $\times 10^{-2}$ inch

TABLE 17

COMPARISON OF DEFLECTION MEASURED IN ACCORDANCE
WITH CGRA & WASHO PROCEDURES (Plotted in Plate 15)

Highway 14

Section	Pavement Thickness		Deflection		Deflection		N
	inch	inch	(WASHO)	inch	(CGRA)	inch	
	Surface	Base	Total	t	Δ	t	Δ
3-13	2½	6	6½	102	5.7	102	6.6
14-17	2.5	9	11.5	95	4.7	95	5.5
18-22	3	9	12	96	3.6	93	4.4
24-25	3	12	13	93	3.5	93	4.6
26-29	4	9	13	99	2.9	99	3.5
30, 33-37	4	12	16	98	2.8	98	3.1
31-32	4	14	18	98	2.2	98	2.6

Nomenclature as shown in Table 16

TABLE 18

DEFLECTION vs CHRONOLOGICAL AGE OF PAVEMENT - I
(Plotted in Plate 16)

Pavement Thickness			Highway 9			Highway 13			Highway 14			Highway 16		
inches														
Surface	Base	Total	A ¹	R ²	Δ ³	A	R	Δ	A	R	Δ	A	R	Δ
2½	6	8½	7	5.0 ⁴	5.2 ⁵	7	5.0	3.7	7	6.0	7.0			
				6.1	3.5		4.8	4.6		6.0	5.4			
				6.6	3.0		3.8	4.2		5.9	5.4			
				6.0	4.3		3.8	2.8		6.1	6.4			
				6.4	4.8		4.6	5.8		5.2	7.6			
				6.0 ₆	5.4		5.1	8.3		6.2	4.1			
				(6.0)	(4.4)		4.9	3.5		7.0	5.7			
							4.1	3.8		6.7	4.8			
							4.0	5.8		6.8	4.9			
							5.2	4.5		6.6	4.9			
							5.4	7.2		7.0	4.9			
							4.5	7.6						
							(4.6)	(5.2)		(6.3)	(5.6)			
2½	9	11½				6	6.6	4.9	6	7.4	5.5			
							6.6	4.9		7.3	5.6			
							6.9	5.7		6.8	5.9			
							(6.7)	(5.2)		7.5	5.4			
										(7.3)	(5.6)			
3	9	12				5	7.0	3.9	3	8.1	4.7	5	8.2	3.3
							7.0	4.4		7.8	3.9		8.2	3.7
							7.3	3.9		7.8	5.0		8.0	4.1
							7.5	3.2		8.0	3.5		(8.1)	(3.7)
							(7.2)	(3.9)		8.4	4.1			
										8.5	5.0			
										(8.1)	(4.4)			
4	9	13				2	8.1	3.5	3	8.3	4.0			
							8.1	3.0		8.4	4.0			
							8.3	2.3		(8.4)	(4.4)			
							(8.2)	(2.9)						
						1	8.3	1.9	2	8.0	3.6			
							8.6	2.1		8.1	2.8			
							8.6	2.2		(8.1)	(3.2)			
							(8.5)	(2.1)						

1 A Denotes Chronological Age of pavement in year

2 R Denotes Performance rating of pavement

3 Δ Denotes Deflection in $\times 10^2$ inch

4 Average values of five determinations

5 Average values of ten Determinations

6 Figures in brackets are mean values

TABLE 19

DEFLECTION vs CHRONOLOGICAL AGE OF PAVEMENT - II
(Plotted in Plate 17)

Route	Sections	Average Deflection x 10 ⁻² inch	Chronological Age of Pavement Years	Traffic Coverage x 100	Number of Trials
13	21 - 23	2.93	2	341	30
	25 - 27	2.07	1	174	30
14	26 - 27	4.0	3	1096	20
	28 - 29	3.2	2	774	20

TABLE 20
VARIABILITY OF DEFLECTIONS MEASURED AT VARIOUS SECTIONS
(Plotted in Plate 18)

Sections	Pavement Thickness			Chronological Age, Years	Deflection x 10 ⁻² in.	Temperature °F
	Surface	Base	Total Inches			
Highway 14						
3	2½	6	8½	7	4.6	108
					8.8	100
					4.4	108
					8.8	101
					8.0	108
					6.4	101
					8.6	108
					8.0	96
					(7.2) ¹	
					(2.0) ²	
26	4	9	13	3	4.2	96
					2.6	92
					3.2	100
					5.2	94
					5.4	100
					5.2	94
					3.0	100
					4.0	95
					3.0	99
					(3.9)	
					(1.0)	
29	4	9	13	2	3.0	96
					2.4	102
					3.6	100
					2.6	103
					2.8	100
					2.0	104
					2.0	100
					3.0	102
					4.0	100
					3.0	102
					(2.9)	
					(0.6)	
32	4	14	18	1	2.8	100
					2.2	97
					2.8	102
					2.0	97
					2.2	101

.....forward

TABLE 20 - Continued

VARIABILITY OF DEFLECTIONS MEASURED AT VARIOUS SECTIONS
(Plotted in Plate 18)

Sections	Pavement Thickness			Chronological Age, Years	Deflection x 10 ⁻² in.	Temperature OF
	Surface	Base	Total Inches			
					2.4	97
					2.6	101
					2.2	96
					3.0	99
					2.6	96
					2.2	99
					2.4	96
					2.8	99
					2.6	94
					2.2	100
					2.6	93
					(2.4)	
					(0.4)	

- 1 Average Deflection
- 2 Standard Deviation of deflection

TABLE 21

DEFLECTION-PAVEMENT THICKNESS-PAVEMENT PERFORMANCE
RATINGS-PAVEMENT CHRONOLOGICAL AGES
(Plotted in Plates 19, 23, 24)

Pavement Thickness inches	Performance Rating of Pavement	DEFLECTION, x 10 ⁻² inch								
		CHRONOLOGICAL AGE OF PAVEMENT, year								
		10	8	7	6	5	4	3	2	1
<u>Highway 16</u>										
8	4.9	7.9								
	5.1	8.8								
	3.2	8.5								
	3.4	8.7								
	4.1	7.5								
	4.3	6.1								
	3.9	6.5								
	3.3	7.3								
	2.7	9.1								
	3.2	8.5								
	2.6	8.1								
	2.4	7.0								
	2.5	8.0								
	2.6	5.6								
	2.6	7.2								
	3.6	7.8								
	3.4	8.5								
	3.6	8.1								
	4.3	8.9								
	(3.5)									
8½	<u>Highway 9</u>									
	5.0					5.2				
	6.1					3.5				
	6.6					3.0				
	6.0					4.3				
	6.4					4.8				
	6.0					5.4				
	<u>Highway 13</u>									
	5.0					3.7				
	4.8					4.6				
	3.8					4.2				
	3.8					2.8				
	4.6					5.8				
	5.1					8.2				

144

TABLE 21 - Continued

Pavement Thickness inches	Performance Rating of Pavement	DEFLECTION, $\times 10^{-2}$ inch								
		CHRONOLOGICAL AGE OF PAVEMENT, year								
		10	8	7	6	5	4	3	2	1
	4.9			3.5						
	4.1			3.8						
	4.0			5.8						
	5.2			4.5						
	5.4			7.2						
	4.5			7.6						
	4.9			8.2						
	<u>Highway 14</u>									
	6.0			7.0						
	6.0			5.4						
	5.9			5.4						
	6.1			6.4						
	5.2			7.6						
	6.2			4.1						
	7.0			5.7						
	6.7			4.8						
	6.8			4.9						
	6.6			4.9						
	7.0			4.9						
	<u>Highway 16</u>									
	5.8			5.1						
	6.3			4.9						
	6.8			4.6						
	6.7			5.0						
	6.5			5.1						
	6.7			6.2						
	7.0		5.7							
	5.8		5.4							
	6.5		7.4							
	5.4		8.5							
	4.6		7.6							
	4.6		7.9							
	5.4		5.5							
	6.1	5.5								
	4.4	6.0								
	6.3	5.1								
	(5.7)									
	<u>Highway 9</u>									
11½	6.6				4.5					
	4.5				3.6					
	<u>Highway 13</u>									
	6.6				4.9					
	6.6				4.9					
	6.9				5.7					

TABLE 21 - Continued

Pavement Thickness inches	Performance Rating of Pavement	DEFLECTION, $\times 10^{-2}$ inch									
		CHRONOLOGICAL AGE OF PAVEMENT, year									
		10	8	7	6	5	4	3	2	1	
	<u>Highway 14</u>										
	7.4				5.5						
	7.3				5.6						
	6.8				5.9						
	7.5				5.4						
	(6.7)										
12	<u>Highway 13</u>										
	7.0					3.9					
	7.0					4.4					
	7.3					3.9					
	7.5					3.2					
	<u>Highway 16</u>										
	8.2					3.3					
	8.2					3.7					
	8.0					4.1					
	<u>Highway 14</u>										
	8.1							4.7			
	7.8							3.9			
	7.8							5.0			
	8.0							3.5			
	8.4							4.1			
	8.5							5.0			
	(7.8)										
13	<u>Highway 9</u>										
	7.3					2.0					
	7.9						2.4				
	<u>Highway 13</u>										
	8.1								3.5		
	8.1								3.0		
	8.3								2.3		
	8.3									1.9	
	8.6									2.1	
	8.6									2.2	
	<u>Highway 14</u>										
	8.3							4.0			
	8.4							4.0			
	8.0								3.6		
	8.1								2.8		
	(8.2)										
14	<u>Highway 16</u>										
	8.6						4.0				
	8.6						3.9				
	8.4						4.2				

TABLE 1

Date		Time		Location		Remarks	

TABLE 21 - Continued

Pavement Thickness inches	Performance Rating of Pavement	DEFLECTION, $\times 10^{-2}$ inch								
		CHRONOLOGICAL AGE OF PAVEMENT, year								
		10	8	7	6	5	4	3	2	1
	8.4						3.0			
	8.5						4.4			
	8.5						3.9			
	(8.5)									
15	<u>Highway 9</u>									
	7.7						2.1			
	8.0						2.2			
	8.0						2.2			
	8.1						2.5			
	7.4						2.6			
	<u>Highway 14</u>									
	8.0							4.6		
	8.0							4.5		
	(7.9)									
16	<u>Highway 14</u>									
	8.3									2.5
	8.5									2.6
	8.7									2.8
	8.7									3.4
	8.5									2.5
	(8.5)									
18	<u>Highway 14</u>									
	8.4								3.0	
	8.6									2.3
	(8.5)									

1. Data are from Highways 9, 13, 14, and 16.
2. Subgrade Soil Type: CL
3. No Re-surfacing
4. Each rating value is average of five determinations.
5. Each deflection value is average of ten determinations.
6. Figures in brackets are the average values.

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42

94

TABLE 22

DEFLECTION AND SUBGRADE SOIL TYPE - I
(Plotted in Plate 20)

Pavement Thickness			R ¹	t ²	SUBGRADE SOIL TYPE				
inches					CH	CI	CL	SP	SC
Surface	Base	Total							
Highway 9									
2½	9	11½	(3.5)	(38)		(4.3)			
			3.8 ³	37	3.8 ⁴				
			2.8	40	4.8				
			2.5	43	4.6				
			2.9	38	4.7				
			4.9	41	3.9				
			5.5	54	2.9				
			(3.7)	(42)	(4.2)				
4	11	15	7.7	33			2.1		
			8.0	34			2.2		
			8.0	34			2.2		
			8.1	51			2.5		
			7.4	51			2.6		
			(6.8)	(41)			(2.3)		
			7.5	34				1.5	
			7.7	38				1.7	
			8.0	35				1.7	
			(7.4)	(36)				(1.6)	
4	9	13	(7.9)	(45)			(2.4)		
			(7.8)	(44)		(2.5)			
3	9	12	(8.1)	(68)				(3.8)	
			(8.0)	(72)			(4.1)		

1 R denotes pavement performance rating

2 t denotes pavement temperature, °F

3 Average of five values

4 Average of ten determinations

Figures in brackets are average values

TABLE 23

DEFLECTION AND SUBGRADE SOIL TYPE - II
(Plotted in Plate 21)

Pavement Thickness inches			Deflection x 10 ⁻² Inch	
Surface	Base	Total	CL	SP
4	9	13	1.8	1.4
			2.6	1.8
			1.8	1.6
			1.8	1.4
			2.4	1.6
			2.4	1.8
			2.0	1.8
			2.8	1.6
			2.6	1.8
			2.2	1.4
			(2.2) ¹	(1.6)
			(0.4) ²	(0.2)

1 Average value

2 Standard Deviation

TABLE

OF THE

REVENUE ACCOUNTS

FOR THE YEAR 1880

AS REPORTED BY THE

COMMISSIONERS OF THE

LAND OFFICE

IN RESPONSE TO A

RESOLUTION OF THE

HOUSE OF REPRESENTATIVES

PASSED MAY 10, 1880

AND CONFIRMED BY THE

SENATE MAY 15, 1880

AND BY THE PRESIDENT

MAY 20, 1880

AND BY THE SUPREME COURT

JUNE 1, 1880

AND BY THE CONGRESS

AND BY THE PRESIDENT

AND BY THE SUPREME COURT

AND BY THE CONGRESS

AND BY THE PRESIDENT

AND BY THE SUPREME COURT

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AND BY THE SUPREME COURT

AND BY THE CONGRESS

AND BY THE PRESIDENT

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AND BY THE CONGRESS

TABLE 24

DEFLECTION AND PAVEMENT TEMPERATURE
(Plotted in Plate 22)

DEFLECTION, x 10 ⁻² inch																						
Temperature, °F																						
58	60	61	72	74	84	85	86	87	88	90	92	93	94	95	96	97	99	100	101	102		
2.2	2.2	2.0	2.2	2.0	2.4	3.0	2.0	2.0	2.4	2.4	2.6	2.6	2.6	3.4	2.2	2.2	3.0	2.8	2.2	2.8		
	2.2	2.2	2.4	2.6	2.2		2.6		1.8	1.6	2.4	2.8	2.8		2.6	2.0	2.2	2.2	2.6			
					2.0		3.0		2.4		2.4	2.4	2.8		2.4	2.4	2.8	3.8				
					1.6		2.0		2.8				2.6		3.4	2.6		2.0				
					2.6				2.4				3.0		2.2	2.6						
					2.6								2.2		2.2							
															2.6							
															2.8							
															3.6							
															2.8							
2.2	2.2	2.1	2.3	2.3	2.2	3.0	2.4	2.0	2.4	2.0	2.5	2.6	2.7	3.4	2.7	2.4	2.7	2.7	2.4	2.8 *		

* Average values
Deflections were taken at section 32 of Highway 14

TABLE 26

DEFLECTION & PAVEMENT RATINGS
(Plotted in Plates 25, 26, 27, & 28)

Route	Pavement Thickness inches			Δ^1	Rating	A ²	
	Surface	Base	Total				
9	2½	6	6½	5.2	5.0	7	Curve A in Plate 25
				3.5	6.1		
				3.0	6.6		
				4.3	6.0		
				4.8	6.4		
	2½	9	11½	5.4	6.0	6	Curve B
				3.8	3.8		
				4.8	2.8		
				4.6	2.5		
				4.7	2.9		
	4	11	13	3.9	4.9	4	Curve C
				2.9	5.5		
				2.1	7.7		
				2.2	8.0		
				2.2	8.0		
13	2½	6	8½	2.5	8.1	7	Curve A in Plate 26
				2.6	7.4		
				3.7	5.0		
				4.6	4.8		
				4.2	3.8		
				2.8	3.8		
				5.8	4.6		
				8.3	5.1		
				3.5	4.9		
				3.8	4.1		
				5.8	4.0		
				4.5	5.2		
				7.2	5.4		
				7.6	4.5		
				8.2	4.9		
	3	9	12	3.9	7.0	5	Curve B
				4.9	7.0		
				3.9	7.3		
	4	9	13	3.2	7.5	1	Curve C
				1.9	8.3		
				2.1	8.6		
14	2½	6	8½	2.2	8.6	7	Curve A in Plate 27
				7.0	6.0		
				5.4	6.0		
				5.4	5.9		
				6.4	6.1		
				7.6	5.2		
				4.1	6.2		

TABLE 26 - Continued

Route	Pavement Thickness inches			Δ^1	Rating	A ²	
	Surface	Base	Total				
16	2½	9	11½	5.7	7.0	6	Curve B
				4.8	6.7		
				4.9	6.8		
				4.9	6.6		
				4.9	7.0		
				5.5	7.4		
				5.6	7.3		
				5.9	6.8		
	4	12	16	5.4	7.5		Curve C
				2.5	8.3		
				2.6	8.5		
				2.8	8.7		
				3.4	8.7		
				2.5	8.5		
				7.9	4.9	10	Curve A in Plate 28
				8.8	5.1		
				8.5	3.2		
				8.7	3.4		
				7.5	4.1		
				6.1	4.3		
				6.5	3.9		
				7.3	3.3		
	5½	3	8½	9.1	2.7	8	Curve B
				8.5	3.2		
				8.1	2.6		
				7.0	2.4		
				8.0	2.5		
				5.6	2.6		
				7.2	2.6		
				7.8	3.6		
	3	11	14	8.5	3.4	4	Curve C
				8.1	3.6		
				8.9	4.3		
				5.7	7.0		
				5.4	5.8		
				7.4	6.5		
				8.5	5.4		
				7.6	4.6		
				7.9	4.6		
				5.5	5.4		
				4.0	8.6		
				3.9	8.6		
				4.2	8.4		
				3.0	8.4		
				4.4	8.5		
				3.9	8.5		

1 Deflection x 10⁻² inch

2 Chronological age of pavement, years

TABLE 27

COEFFICIENT OF VARIATION OF RATING-SECTION LENGTH-
CHRONOLOGICAL AGE OF PAVEMENT
(Plotted in Plates 29 & 30)

Section	Length mi.	COEFFICIENT OF VARIATION OF RATING, $\frac{s}{\bar{X}}$, %										
		1 \bar{X}	2 σ	CHRONOLOGICAL AGE OF PAVEMENT, years								
				10	8	7	6	5	4	3	2	1
Highway 9												
1	22.5	5.0	0.5				10.0					
2	2.0	6.1	0.5				8.2					
3	2.1	6.6	0.6				9.1					
4	0.9	6.0	0.6				10.0					
5	0.8	6.4	0.6				9.4					
6	10.0	6.0	0.3				5.0					
7	2.1	6.6	0.6					9.1				
8	4.0	4.5	0.4					8.9				
9	4.4	3.5	0.4					11.4				
10	5.8	3.8	0.5					13.2				
11	2.6	2.8	0.5					17.9				
12	1.7	2.5	0.5					20.0				
13	6.9	2.9	0.7					24.1				
14	1.6	4.9	1.0					20.4				
15	0.4	5.5	0.7					12.7				
16	0.9	7.3	0.7						9.6			
17	0.3	6.5	0.7							10.8		
18	2.0	7.1	0.5							7.1		
19	8.4	7.4	0.5							6.8		
20	0.3	7.4	0.5							6.8		
21	2.1	7.1	1.0							14.1		
22	0.8	7.6	0.9							11.8		
23	0.9	7.5	0.9							12.0		
24	0.3	7.7	0.7							9.1		
25	0.4	8.0	0.8							10.0		
26	1.4	7.7	1.1							14.3		
27	1.8	8.0	0.5							6.3		
28	0.9	8.0	0.6							7.5		
29	0.4	8.1	0.6							7.4		
30	2.8	7.4	0.9							12.2		
31	1.0	7.6	0.8							10.5		
32	0.8	7.5	0.7							9.3		
33	3.5	7.9	0.9							11.4		
34	1.8	7.9	0.9							11.4		
35	0.2	7.8	0.9							11.5		

TABLE 27 - Continued

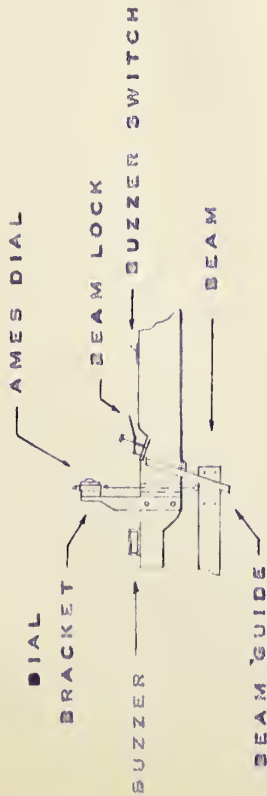
					COEFFICIENT OF VARIATION OF RATING, $\frac{\sigma}{\bar{X}}$, %								
					CHRONOLOGICAL AGE OF PAVEMENT, years								
Section	Length mi.	\bar{X}	σ		10	8	7	6	5	4	3	2	1
<u>Highway 16</u>													
1	5.8	6.6	0.8	12.1									
2	2.8	6.8	0.5	7.4									
3	1.0	5.8	0.6	10.3									
4	0.6	4.9	0.9	18.4									
5	0.5	5.1	0.7	13.7									
6	1.9	3.2	0.6	18.8									
7	1.8	3.4	0.4	11.8									
8	0.4	4.1	0.5	12.2									
9	0.8	4.3	1.2	27.9									
10	1.5	3.9	0.6	15.4									
11	1.1	3.3	0.7	21.2									
12	0.3	2.7	0.8	29.6									
13	1.9	3.2	0.6	18.8									
14	3.8	2.6	0.5	19.2									
15	1.0	2.4	0.7	29.2									
16	0.8	2.5	0.7	28.0									
17	0.9	2.6	0.7	26.9									
18	1.1	2.6	0.7	26.9									
19	1.0	3.6	1.0	27.8									
20	2.2	3.4	0.7	20.6									
21	1.4	3.6	0.9	25.0									
22	2.0	4.3	0.6	14.0									
23	4.7	7.5	0.4			5.3							
24	0.8	8.0	0.3			3.8							
25	0.2	8.4	0.6			7.1							
26	1.6	8.3	0.6			7.2							
27	3.6	8.4	0.6			7.1							
28	0.8	8.4	0.6			7.1							
29	3.4	8.5	0.6			7.1							
30	0.7	8.5	0.5			5.9							
31	2.9	8.4	0.6			7.1							
32	1.3	8.4	0.7			8.3							
33	4.4	8.6	0.8			9.4							
34	2.4	8.2	0.8			9.8							
35	8.7	8.6	0.7							8.1			
36	2.7	8.6	0.9							10.5			
37	6.9	8.4	0.8							9.5			
38	0.7	8.4	0.9							17.1			
39	4.6	8.5	0.7							8.2			
40	0.5	8.5	0.7							8.2			

TABLE 27 - continued

COEFFICIENT OF VARIATION OF RATING, $\frac{\sigma}{\bar{X}}$, %												
CHRONOLOGICAL AGE OF PAVEMENT, years												
Section	Length mi.	\bar{X}	σ	10	8	7	6	5	4	3	2	1
41	2.5	8.2	0.7					8.5				
42	1.5	8.2	0.5					6.1				
43	10.8	8.1	0.8					9.9				
44	2.9	8.1	0.6					7.4				
45	0.9	8.2	0.9					11.0				
46	2.3	8.0	1.0					12.5				
47	10.0	5.8	1.7			29.3						
48	0.6	6.3	1.1			17.5						
49	0.8	6.8	1.1			16.2						
50	1.7	6.7	1.4			20.9						
51	0.7	6.5	1.5			23.1						
52	5.9	6.7	1.1			16.4						
53	3.4	7.0	0.9		12.9							
54	1.1	5.8	1.6		27.6							
55	0.8	6.5	1.5		23.1							
56	2.1	5.4	1.5		27.8							
57	0.8	4.6	1.1		23.9							
58	4.0	4.6	0.6		13.1							
59	0.8	5.4	0.7		13.0							
60	0.3	6.1	1.0	16.4								
61	1.1	4.4	0.8	18.2								
62	14.0	6.3	0.7	11.1								

1. Mean of five rating values.
2. Standard deviation of rating.

APPENDIX II



ENLARGED SIDE VIEW
OF CONTROL AREA

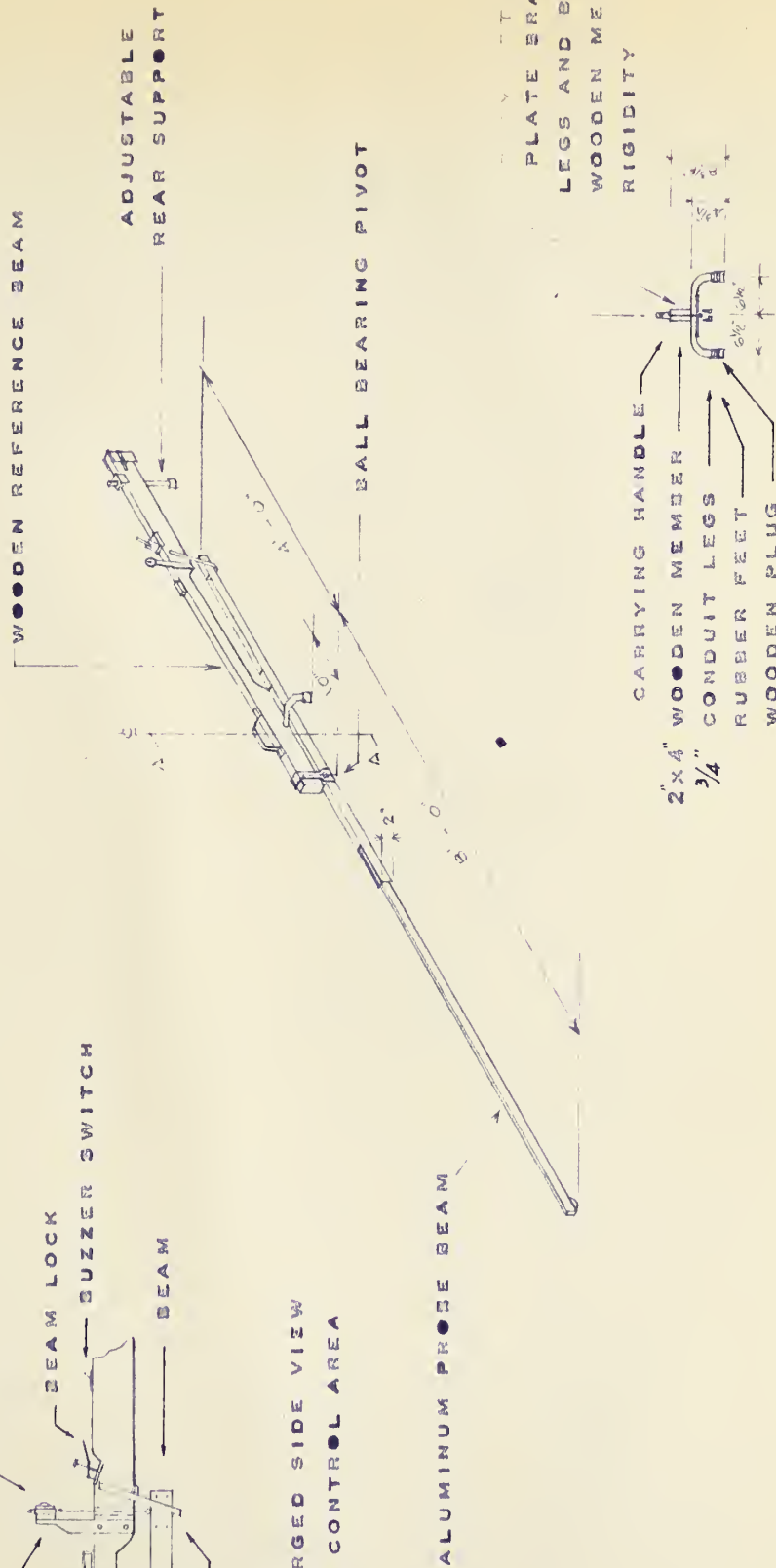
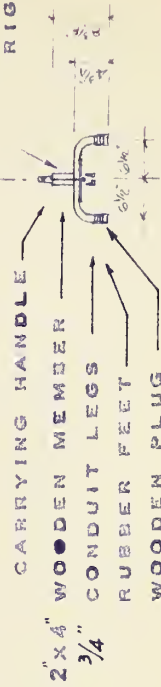
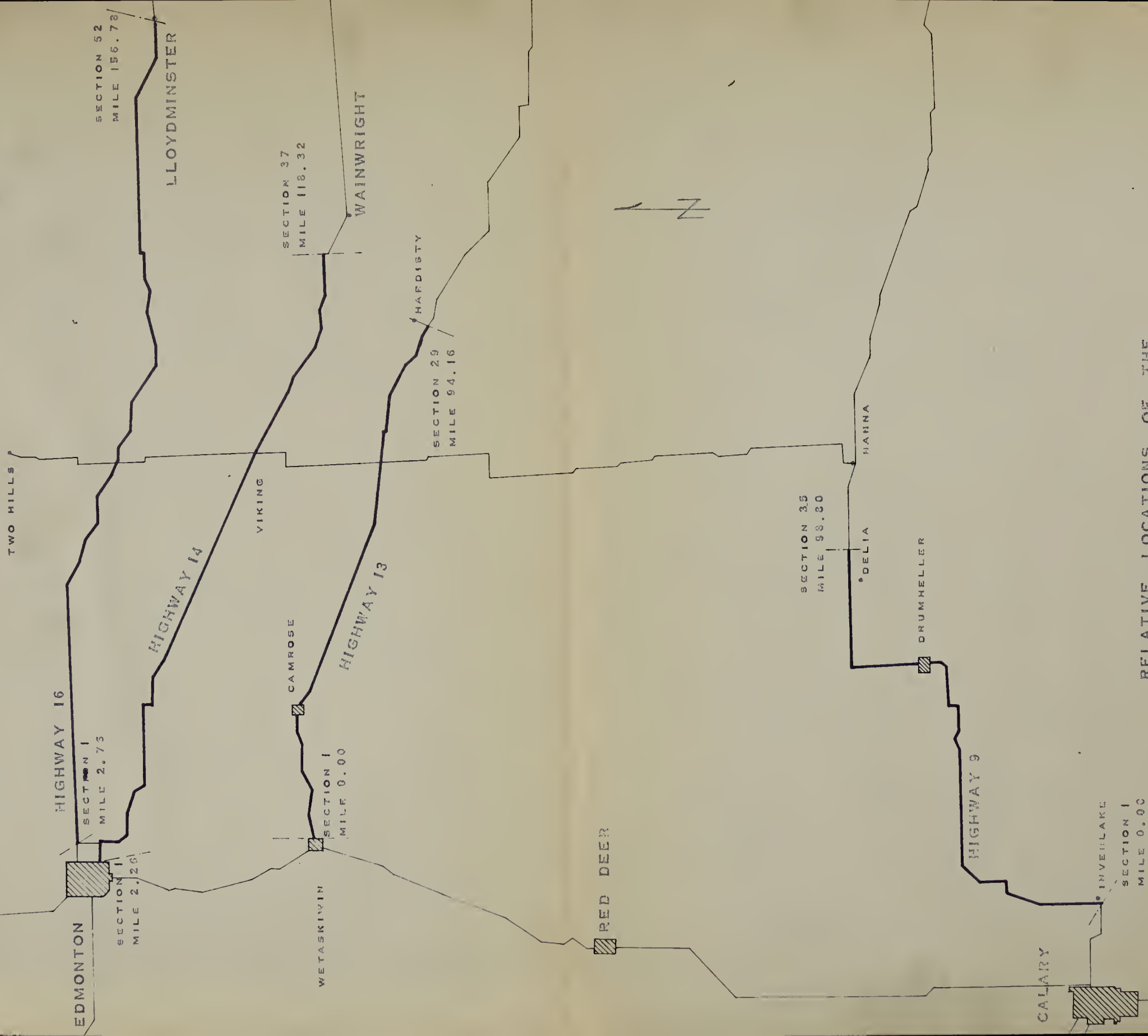


PLATE BRAZED TO
LEGS AND BOLTED TO
WOODEN MEMBER FOR
RIGIDITY



SECTION A-A



RELATIVE LOCATIONS OF THE
FOUR HIGHWAYS INVESTIGATED

SCALE 1 INCH = 16 MILES

APPENDIX III

SAMPLE CALCULATIONS

1. Calculation of pavement deflection - Total Pavement

Thickness Curve in Plate 1

$$\Delta = \frac{1.5 \text{ pr}}{E_2} F_w$$

$$P = 80 \text{ psi}$$

$$h = 6''$$

$$\frac{E_2}{E_1} = \frac{2,000}{60,000} = \frac{1}{30}$$

$$F_w = 0.266 \text{ @ } h = 9''$$

$$\begin{aligned} \Delta &= \frac{1.5 \text{ pr}}{E_2} F_w \\ &= \frac{1.5 \times 80 \times 6}{2,000} \times 0.266 \\ &= .0958'' \end{aligned}$$

2. Calculation of Pavement Deflection - Modulus Ratio Curve in Plate 2

$$F_w = 0.38 \text{ @ } h = 15'' \text{ and } \frac{E_2}{E_1} = \frac{1}{5}$$

$$\Delta = \frac{1.5 \text{ pr}}{E_2} F_w$$

$$\text{For } E_2 = 2,000 \text{ psi}$$

$$\Delta = \frac{1.5 \times 80 \times 6}{2,000} \times 0.38 = 0.137''$$

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3. Calculation of Modulus of Subgrade - Modulus of the Reinforced layer curve in Plate 3

$$F_w = 0.49 @ h = 5'' \text{ and } \frac{E_2}{E_1} = \frac{1}{5}$$

$$E_2 = \frac{1.5 \text{ pr}}{\Delta} F_w$$

$$\text{For } \Delta = 0.05''$$

$$\begin{aligned} E_2 &= \frac{1.5 \times 80 \times 6}{.05} = 0.49 \\ &= 7060 \text{ psi} \end{aligned}$$

4. Derivation fo Empirical Equations in a form of

$$\frac{E_1}{E_2} = \frac{1}{Ah-B}$$

$$\text{For } \Delta = .05$$

$$E_2 = 2,000 \text{ psi}$$

$$\begin{aligned} F_w &= \frac{E_2}{1.5 \text{ pr}} \\ &= \frac{0.05 \times 2000}{1.5 \times 80 \times 6} = 0.139 \end{aligned}$$

$$\frac{r}{h} = 2.036 @ F_w = 0.139 \text{ and } \frac{E_1}{E_2} = 70$$

$$\begin{aligned} \therefore h &= 2.036 \times r \\ &= 2.036 \times 6 \\ &= 12.2 \times 6 \end{aligned}$$

Taking a functional form of $y = \frac{1}{ax-b}$ for the relationship between the variables considered.

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DEPARTMENT OF CHEMISTRY

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The "normal" equations for evaluating the values of
a and b are

$$(I) \quad \sum \left(\frac{1}{y} \right) = Na + b \sum (x)$$

$$(II) \quad \sum \left(\frac{x}{y} \right) = a \sum (x) + b \sum (x^2)$$

Substituting and solving

$$a = 0.00271$$

$$b = 0.0197$$

$$\text{Thus } \frac{E_1}{E_2} = \frac{1}{0.00271 h - 0.0197}$$

5. Calculation of Deflection - Fire Pressure curve in Plate 10

$$r = \sqrt{\frac{A}{\pi}}$$

$$= \sqrt{\frac{9000}{80 \pi}}$$

$$= 5.98''$$

$$\frac{h}{r} = 1.504 \text{ for } h = 9''$$

$$\text{For } \frac{E_2}{E_1} = \frac{4,000}{120,000} = \frac{1}{30}$$

$$F_w = .252$$

$$\begin{aligned} \Delta &= \frac{1.5 \text{ pr}}{E_2} F_w \\ &= \frac{1.5 \times 80 \times 6}{4,000} \times .252 \\ &= 0.0455 \end{aligned}$$

1. The first part of the paper is devoted to a general discussion of the problem.

2. The second part of the paper is devoted to a detailed analysis of the results.

3. The third part of the paper is devoted to a discussion of the conclusions.

4. The fourth part of the paper is devoted to a discussion of the conclusions.

5. The fifth part of the paper is devoted to a discussion of the conclusions.

6. The sixth part of the paper is devoted to a discussion of the conclusions.

7. The seventh part of the paper is devoted to a discussion of the conclusions.

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21. The twenty-first part of the paper is devoted to a discussion of the conclusions.

22. The twenty-second part of the paper is devoted to a discussion of the conclusions.

23. The twenty-third part of the paper is devoted to a discussion of the conclusions.

24. The twenty-fourth part of the paper is devoted to a discussion of the conclusions.

25. The twenty-fifth part of the paper is devoted to a discussion of the conclusions.

APPENDIX IV

APPLICATION OF STATISTICAL METHODS OF ANALYSIS

The arithmetic mean is used as the average of many observations on one variable and the standard deviation, as the measure of the extent to which the observations scatter about the mean. For N observations $X_1, X_2 \dots X_N$, the arithmetic mean is:

$$\bar{X} = \frac{\sum (X)}{N} ;$$

and the standard deviation is the root mean-square of the N deviations from the mean:

$$\sigma = \sqrt{\frac{\sum (X - \bar{X})^2}{N}}$$

The coefficient of variation is to relate the measure of dispersion to its average and to convert it to percentage form. It is the standard deviation divided by the arithmetic mean,

$$V = \frac{\sigma}{\bar{X}} 100$$

For the analysis of the relationship between two variables, a functional relation is first assumed and the values of the constants in the assumed equations are determined by the method of least squares. The principle of least squares states that a line of best fit to a series of values is a line, the sum of the squares of the deviations about which will be a minimum. The equations used for the analysis of the functional relationship in this dissertation include:

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linear, $y = mx + b$

hyperbola, $y = \frac{1}{mx - b}$ and $y = \frac{1}{x}$

parabola, $y = a + bx + cx^2 + dx^3 + ex^4$

B29788